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## Remarks Concerning a Paper by Wilker<sup>1)</sup>

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(20. XII. 1951.)

WILKER states that,  $W(1)$ <sup>2)</sup>,

$$\frac{dr}{dt} = \frac{\partial r}{\partial t} + [r, H] \quad (1)$$

is invariant with respect to a homogeneous canonical transformation only for the trivial case of a free particle.

WILKER deduces this result by using the homogeneous canonical formalism,  $W(2)$ ,  $W(3)$ ,

$$q'_i = \frac{\partial \mathfrak{S}}{\partial p_i}, \quad P'_i = -\frac{\partial \mathfrak{S}}{\partial q_i}; \quad \mathfrak{S}(q_i, p_i) = 0 \quad (2)$$

$$[i = 1, \dots, f+1]$$

$$r' = \{r, \mathfrak{S}\} \quad (3)$$

$$\mathfrak{S} \equiv t'(\tau) (H + p_t) \quad (4)$$

WILKER shows,  $W(4)$ , that, if  $r = r(q_i, t, p_i, p_t)$

$$\frac{dr}{dt} = \frac{\partial r}{\partial t} + [r, H] - \frac{\partial H}{\partial t} \frac{\partial r}{\partial p_t} \quad (5)$$

WILKER now concludes that (1) follows from (5) only if  $\partial H/\partial t = 0$ .

Now (1) is usually understood to apply in the formalism that can be LORENTZ invariant, but that is not LORENTZ invariant in appearance. In this formalism  $r$  is expressed as a function of  $q_i$ ,  $p_i$ ,  $t$ . If  $r$  is a function of  $H$  as well (or any other quantity, e.g. angular momentum),

$$\frac{dr}{dt} = \frac{\partial r}{\partial t} + [r, H] + \frac{\partial r}{\partial H} \frac{\partial H}{\partial t} \quad (6)$$

where  $dH/dt = \partial H/\partial t$ .

(6) is equivalent to WILKER's  $W(4)$ , (5) above, because  $\mathfrak{S} = 0$ , and  $p_t = -H$ . However, the third term on the right side of (6) will

<sup>1)</sup> P. WILKER, *Helv. Phys. Acta*, 319, **24**, 1951.

<sup>2)</sup>  $W(1)$  etc. refers to WILKER's paper.

not appear if  $H$  is expressed as a function of  $q_i, p_i, t$ . (6) therefore reduces to (1) by arranging that  $\partial r/\partial H = 0$ , and not by using WILKER's stronger condition  $\partial H/\partial t = 0$ .

If  $r = r(q_i, p_i, t)$  in one inertial system, and we carry out a LORENTZ transformation, the Hamiltonian will appear in the function. For a scalar  $r$

$$r(q_i, p_i, t) = \bar{r}(\bar{q}_i, \bar{p}_i, \bar{H}, \bar{t}) \quad (7)$$

where the bar quantities refer to the new inertial system.  $\bar{H}$ , however, can be expressed as a function of  $\bar{q}_i, \bar{p}_i, \bar{t}$ , and, if we substitute  $\bar{H}$  expressed as a function of  $\bar{q}_i, \bar{p}_i, \bar{t}$  in  $\bar{r}$ , the expression (1) will also hold in the new inertial system, because  $\partial \bar{r}/\partial \bar{H} = 0$ .

It may also be remarked here that the homogeneous canonical formalism can be developed very simply by relating the formulation that is non-covariant in appearance

$$\delta \int L\left(x_i, \frac{dx_i}{dt}, t\right) dt = 0$$

with the covariant formulation

$$\delta \int \mathfrak{L}\left(x_\mu, \frac{dx_\mu}{d\tau}\right) d\tau = 0$$

in the following way:

$$L\left(x_i, \frac{dx_i}{dt}, t\right) dt = \mathfrak{L}\left(x_\mu, \frac{dx_\mu}{d\tau}\right) d\tau. \quad (8)$$