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Further Investigations on the Plural Production of Meson Showers

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Summary: Calculations on the size of nucleon initiated meson showers have been extended by including secondary effects of recoil nucleons. It is shown that showers with up to 30 or 40 mesons occur still with reasonable probability in the picture of plural production. The protons accompaying the shower and the angular distribution are discussed. The production of showers by π-mesons is considered. Assuming that these are primarily due to the scattering of the meson by a nucleon it is shown that for high meson energies meson showers are produced. The cross section is estimated to be 0.4—0.9 of the geometrical cross section for heavy elements, and the most important energies. It is found that all the conspicuous features of penetrating showers can be understood by the picture of plural production.

1. Introduction.

In a recent paper[1]) (in the following quoted as I) we have studied the plural production of mesons during the passage of a fast nucleon through a nucleus on the assumption that in an individual nucleon-nucleon encounter only one meson is produced. This was done in order to decide whether any evidence for genuine multiple processes can be derived from the existing experimental material concerning meson showers. It was shown in I that the observed size distribution of meson showers can be quantitatively accounted for by the picture of pure plural production making physically sound assumptions about the individual cross section for meson production. This applies also to "large showers" up to about 20 particles (in Ag or Br) for which the theory gives probabilities in agreement with experiment. In I only the mesons produced be the primary nucleon itself were considered, but allowance was made for recoil nucleons leaving the nucleus. It was assumed that there is about one recoil nucleon for each meson produced and these were included in the shower size as "relativistic particles". As was already pointed out in I, one must expect a further increase in shower size from the meson production of these recoil nucleons. In fact fast nucleons are unable to leave the nucleus. The effect of meson production by recoils will be that the latter are pushed to a lower energy region and instead more mesons will appear. We shall see that for showers of small and medium energies the total shower size (relativistic particles) will remain roughly the same as in I. For the more energetic showers, however, where the recoil nucleons can produce several mesons, the shower size will be increased very considerably. It might, in extreme cases, even be necessary to consider tertiary processes etc., leading to a cascade of meson production inside the same nucleus.

Meanwhile a few photographs of very large showers have been published[2]) containing 30 and more relativistic particles. It is not known how frequent these events are. On the other hand the probability for very large showers as derived in I from the primary effects alone decreases very rapidly with the shower size for very large showers and probably yields a probability far too small to be compatible even with the few cases found.

In view of this situation it seems desirable to estimate the contribution to the shower size from the cascade effects. A calculation on the lines of I is of the same degree of difficulty as the fluctuation problem in ordinary cascade theory. And there is a further physical complication:

In an electron-cascade there is always an unlimited number of atoms available at which Bremsstrahlung can be emitted or pairs created. This is not so in the nucleon cascade under consideration. Consider a very fast primary nucleon passing through nuclear matter. In the first encounter with a nucleon a meson plus a fast recoil nucleon will be produced. For the very high energies we are concerned with the recoil nucleon will often form such a small angle with the direction of the primary that the two will travel practically in the same direction. The second nucleon at rest will be hit then by both the primary and the first recoil nucleon practically at the same time whereas the neighbour nucleons will not be hit directly. It is clear that in this encounter two mesons will be produced but *only one* further recoil nucleon. This comes to the same as saying that no tertiary recoil nucleons are produced. On the other hand the recoil energy transferred to the second nucleon by the simultaneous hit of the primary and secondary nucleon will be greater than if the primary were alone present. This *increases* again the meson producing power of the second recoil nucleon.

In the following we shall try to estimate the number of secondary mesons by a crude but simple model. We shall include only mesons in the "shower size" and leave the discussion of the fast protons to section 3. It will be seen that the probabilities for smallish and medium sized showers are much the same as in I (and in agreement with the measurements) but that very large showers occur much more frequently. In the later sections we shall consider the possibility of shower production by π-mesons and the angular distribution of the mesons produced.

2. Estimate of the number of secondary mesons.

We make the following assumptions:

(i) In a collision of a fast nucleon with energy E with a nucleon at rest one meson is produced. The primary loses the energy σE and a recoil nucleon is produced with energy αE. Obviously $\alpha < \sigma$, and $(\sigma - \alpha) E$ is the energy of the meson.

(ii) Only secondary processes but no tertiary processes are taken into account so that the limitation of the number of nucleons discussed above is taken into account, perhaps in a somewhat exaggerated way. We shall assume that the secondary nucleons produce mesons in the same way as the primary, which implies that in a simultaneous hit of one nucleon by two fast nucleons two mesons are produced. We neglect, though, the increase of recoil energy produced by the recoil nucleons. Clearly this leads rather to an underestimate of the number of secondary mesons.

(iii) Otherwise the same assumptions are made as in I, i. e. that meson production ceases when E becomes less than a certain critical energy E_c which is assumed to be of the order of magnitude of 10^9 e. v.

(iv) To start with the fluctuations will be neglected. We shall first work out the effects for an infinitely thick layer of nuclear matter and later, when the finite size of the nucleus is taken into account, a way will also offer itself to account for the fluctuations, at least roughly.

Let N be the number of nucleons per cm³ and φ the total cross section. After a passage through a distance x in nuclear matter the primary (energy E_0) has made

$$n = N \varphi x$$

encounters with nucleons. The primary has then energy $(1-\sigma)^n E_0$, and n recoil nucleons are produced. These have all the same energy, namely $\alpha (1-\sigma)^{n-1} E_0$. If $(1-\sigma)^{n-1} E_0 \geqslant E_c$ the primary has produced n mesons. The recoil nucleons produce mesons so long as $\alpha (1-\sigma)^{n'-1} E_0 \geqslant E_c$ and the number of mesons produced by them is $n' (n' + 1)/2$.

Thus the total number of mesons produced is

$$\nu = n + \frac{n'(n'+1)}{2},$$

$$(1-\sigma)^{n-1} \geqslant \frac{E_c}{E_0} \qquad \alpha\,(1-\sigma)^{n'-1} \geqslant \frac{E_c}{E_0}. \tag{1}$$

The two numbers n, n' will, of course, not coincide, and $n' < n$. n is also the number of recoil nucleons existing after n collisions of the primary and these have all energies $\leqslant E_c$. We shall discuss them in section 3.

(2) is only true so long as the length of passage through the nucleus x required for n collisions is less than the size of the nucleus. It will be seen that the finite size of the nucleus has a cutting off effect only for large showers, which we can easily take into account later. We shall therefore first work out the secondary effects for an infinitely large nucleus.

The incident spectrum is $\gamma E_c^\gamma \, dE_0/E_0^{\gamma+1}$ or the integral spectrum

$$w(\geqslant E_0) = \left(\frac{E_0}{E_c}\right)^\gamma, \quad \gamma \sim 1.5.$$

Since both n and n' depend on E_0 only (when the constants σ, α are given) the normalized number of showers with ν or more mesons is

$$P(\geqslant \nu) = e^{-\gamma y}, \quad y = \log\left(\frac{E_0}{E_c}\right) \tag{2}$$

and the relation between y and ν is given by (2)

$$n - 1 = \frac{y}{\log \frac{1}{1-\sigma}}, \qquad n' - 1 = \frac{y - \log 1/\alpha}{\log \frac{1}{1-\sigma}}. \tag{3}$$

From (1), (2) and (3) y could be expressed by ν and therefore $P(\geqslant \nu)$ by ν, but it is more convenient to use the parametric representation (3). Before we work out $P\,(\geqslant \nu)$ a minor alteration is advisable: It is likely that only 2/3 of the mesons produced are charged. Then the total number of charged mesons in the shower is

$$\nu = \frac{2}{3}\, n + \frac{n'(n'+1)}{3}. \tag{4}$$

The influence of the secondary processes depends now on the constants σ, α. σ was already determined approximately from the absorption of fast (meson-producing) nucleons[3]) and it was found that $\sigma \sim 1/4$. We can determine σ also so that for the primary mesons only, the results of our present model agree with the exact calculations of I. It is sufficient to do this for an infinitely thick layer of nuclear matter. Then, in the notation of I, $a_A = N\varphi d_A \to \infty$ (d_A = nuclear diameter) and the probability for a shower of n primary mesons becomes simply (cf. equ. (17) of I)

$$P_n = \omega_{\gamma+1}^{n-1}(1-\omega_{\gamma+1})$$

or

$$P(\geqslant n) = \omega_{\gamma-1}^{n-1}$$

Here $\omega_{\gamma+1}$ is defined as

$$\omega_{\gamma+1} = \frac{1}{\varphi}\int_0^E \Phi\left(\frac{\varepsilon}{E}\right)\left(1-\frac{\varepsilon}{E}\right)^{\gamma}\frac{d\varepsilon}{E}.$$

Our present model gives ((2), (3))

$$P(\geqslant n) = (1-\sigma)^{\gamma(n-1)}.$$

Thus σ should be determined so that

$$(1-\sigma)^{\gamma} \to \omega_{\gamma+1}. \qquad (4')$$

The difference between the exact calculation of I and the present model lies then solely in the change of the constant $\omega_{\gamma+1}$. $\omega_{\gamma+1}$ is the average of $(1-\varepsilon/E)^{\gamma}$ whereas $(1-\sigma)^{\gamma}$ is the average of $1-\varepsilon/E$, raised to the power γ. The difference between these two averages is very small. Now it was shown in I that the value of $\omega_{\gamma+1}$ which renders the number of showers with moderate n's well, is about $\omega_{\gamma+1} \simeq 2/3$.

Then (4′) gives also $1-\sigma \sim 0.76$, $\sigma \sim 1/4$.

Next we have to make some assumption about α. It will be reasonable to assume that for the high energies we are concerned with the energy lost is divided up in equal parts by the meson and recoil nucleon. Thus $\alpha \sim 1/8$.

We have then from (1)—(3):

$$n = 1 + 3.5\,y\,, \qquad n' = 3.5\,y - 6.3 \qquad \log P(\geqslant \nu) = -\gamma y \qquad (5)$$

$$\nu = \frac{2}{3}n + \frac{n'(n'+1)}{3}$$

We have also used smaller values of α (for instance $\alpha \sim 1/10$) and it turns out that the results are not very sensitive to α. In Fig. 1. $\log_{10} P(\geqslant\nu)$ is plotted against ν (curve ν (∞)). The dotted straight line gives n (= number of nuclear encounters or number of mesons, including neutretto's, produced by primary. We see the enormous enlargement of the showers for large values of n, through the secondary processes.

The above results have now to be corrected for the finite size of the nucleus. When the nucleus has diameter d_A, a sharp drop of the probability would take place at $n = N\varphi d_A = a_A$, because the primary cannot make more than a_A nuclear collisions, and any finite probability found for $n > a_A$ is due to fluctuations. Similarly, for the secondary mesons we should break off at $1 + n' = a_A$ because $n' + 1$ encounters are required if the recoil nucleons (no matter at which place they are created) should make n' or $n' - 1\ldots$ meson producing collisions.

We had seen in I that, for Ag or Br, the value of a_A which is to be taken to render the distribution including large showers (i. e. up 20 particles) well was about $a_A = 9$. Since now secondary mesons contribute considerably to the shower size we should choose a smaller value (as was already anticipated in I). A smaller value for the total cross section φ is also advocated by the following reason: In reference[3]) a relation was established between $\omega_{\gamma+1}$ and φ, namely $\varphi(1-\omega_{\gamma+1}) = 1.15\left(\frac{\hbar}{\mu c}\right)^2$. Since $N\left(\frac{\hbar}{\mu c}\right)^2 d_A = 1.9$ for Ag or Br, we get, if we choose $\omega_{\gamma+1} = 2/3$, $\varphi = 3.5\left(\frac{\hbar}{\mu c}\right)^2$ or $a_A \sim 7$. Therefore we must expect a more or less sharp drop of the dotted curve marked $n\,(\infty)$ in fig. 1 when $n > 7$. We see that the effect of the finite size of the nucleus becomes very serious really only for comparatively large n. In fig. 1, we have plotted also the exact curve for n, according to I, (curve n(I)) for the constants now used, i. e. $\omega_{\gamma+1} = 0.7$, $a_A = 7$.

It is now likely that a similar decrease, owing to the finite thickness of the nucleus, will take place for the secondary mesons, in the following way: The number of encounters made by secondary nucleons was called n'. In the present model $n' + 1$ would evidently be limited to a_A. Thus we should expect no considerable (or only a small) decrease of n' up to $n' \simeq 6$. Then n' will be gradually diminished, presumably in a similar way as n is. However, $n' = 6$ means already 14 charged secondary mesons. The finite size of the nucleus limits therefore the number of secondary mesons in a much less stringent way than the primary mesons, and only for extremely large showers. If we assume that $n' + 1$ is diminished in the same way as n is, for $n' > 6$, we can estimate the total size of the shower. In this way the curve for ν shown in fig. 7 was obtained. This, of course, is only a crude estimate, but the general trend is quite clear. The departure of ν from $\nu\,(\infty)$ is in the first place due to the difference of n and $n\,(\infty)$ whilst n' is still < 6 and the number of secondary mesons not much affected, although they contribute considerably. From $\nu = 20$ upwards the departure of ν from $\nu(\infty)$ becomes bigger owing to the decrease of n'. It is therefore only the last part of the curve which is in serious doubt. For such high energies also tertiary effects etc. may not be negligible, as discussed above.

The curve for ν agrees with the experiments quoted in I about as well as the theory of I which was based on the primary effects only, but including fast protons as shower particles and using a slightly larger value for a_A*). A slight variation of the constants could make the fit perfect but the inaccuracy of the experiments (especially for large ν) and the theory does not warrant this. (The exp. points for $\nu = 19, 20, 21$ rest on one shower.)

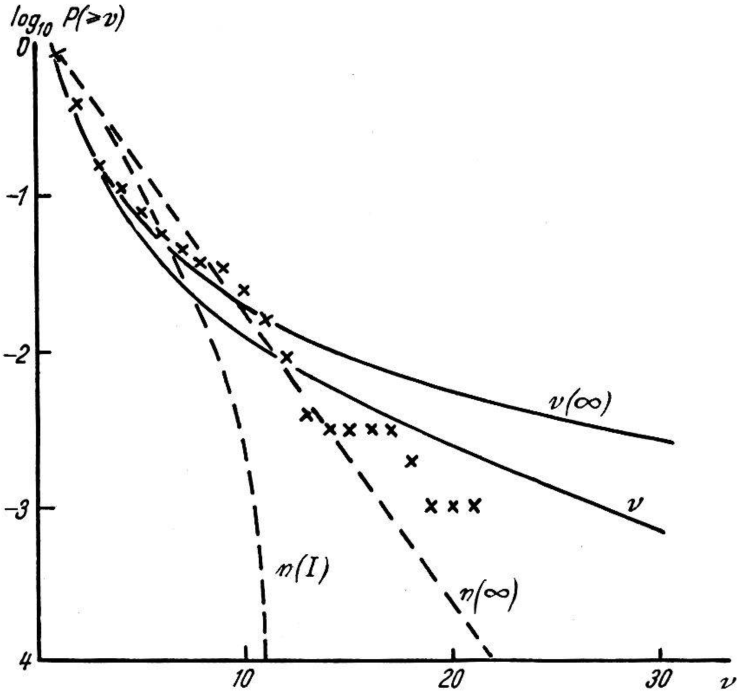

Fig. 1.

Probability P for a shower with more than ν relativistic particles: curve n(I): number of nuclear encounters or primary mesons (including neutretto's, according to I (exact) for Ag or Br; $n(\infty)$: the same for an infinitely thick layer of nuclear matter; curve $\nu(\infty)$: total number of charged primary and secondary mesons for infinitely thick nucleus; curve ν: the same for finite nucleus Ag or Br (crude estimate). x experimental points, ref.[4]), normalized to agree with ν-curve for $\nu = 3$.

The curve for ν should, of course, not be taken as a quantitative prediction (the uncertainties of the constants are too great) but our considerations show that also showers with 30 or 40 charged mesons are to be expected from the picture of plural production with a reasonable probility.

*) The experimental points in Fig. 1 have been corrected by subtracting an estimated contribution from light nuclei (C, N, O). The latter is less than 25%.

Showers have also been observed which were initiated by heavy nuclei[2][9]. On the average one expects that the number of mesons is multiplied by the number of nucleons in the primary particle as compared with a shower initiated by a single nucleon, provided that we compare showers with the same energy of the primary *per nucleon*. So the shower of reference[9] with 56 particles initiated by a very fast α-particle constitutes a shower of 14 per nucleon. If the energy of the primary per nucleon is only of the order E_c (i. e. $\sim 10^9$ e. v.) the result will be that no, or only very few mesons are produced, and we obtain only a very large evaporation star. The point is nicely illustrated in the two showers published by LEPRINCE-RINGUET et al.[2].

In the discussion of individual showers, however, a correlation between shower size and primary energy does not hold in the same way as for the average. We must not forget that the length of path through the nucleus can be much smaller than the average and therefore cases may occur where very energetic showers are quite small. The energy shows up in the angular distribution. So we may expect occasionally narrow showers with comparatively few particles while in the majority of cases a shower with the same number of particles will have a wide angular spread. An example will be discussed in section 5.

3. Protons as shower particles.

In the above estimate protons are not included as shower particles. As we have now also taken into account the meson production by the secondary nucleons the latter will be pushed to a region of lower energy. A nucleon will find it difficult to escape from the nucleus, unless its energy has decreased to a value $< E_c$, which quantity must be expected to be of the order of 10^9 e. v. On the other hand, for lower energies nucleon-nucleon scattering is important. The cross section for scattering is known to be $0{\cdot}9 \cdot 10^{-25}$ cm^2 for $E = 10^8$ e. v. and decreases with E. It is likely that the decrease is roughly $\sim 1/E$, also in the relativistic region and on almost any theory[5]. Then at 10^9 e. v. the scattering cross section has decreased to the order of 10^{-26} cm^2, whereas the cross section for meson production is increasing but has not reached yet its full value (for $E \gg E_c$) which, from our results is probably $6 \cdot 10^{-26}$ cm^2 or so. Thus, somewhere in the region of, say, $5 \cdot 10^8 - 10^9$ e. v. the nucleus is comparatively transparent for nucleons with a total cross section per nucleon not much higher than $1\text{–}2 \cdot 10^{-26}$ cm^2. With a cross section of this magnitude a nucleon would suffer on the average just about 1 or 2 collisions in a diametrical passage through an Ag-nucleus. Nucleons reaching this energy region somewhere within the nucleus will have a good chance to leave the nucleus more or less unhindered. Once, however, a nucleon has reached a lower energy, it will lose further energy by scattering and drag with it a number of low energy nucleons which will gradually go over into the evaporation nucleons. A sharp distinction between "fast protons" and evaporation nucleons is, of course, artificial.

This picture agrees well with the recent results of FOWLER[6]), who finds that practically no protons in showers have energy $> 10^9$ e. v. (whereas mesons of these energies are frequent), and most have energies $< 5 \cdot 10^9$ e. v.

The number of "fast nucleons" follows at once from our model. If we assume that half of them are neutrons and if we include the primary, the number of relatively fast protons, which we must expect to have energies in the "transparency region" is

$$\nu_P = \frac{1}{2} + \frac{n}{2}.$$

Thus, for example, a shower with 5, 10, 20 charged mesons would be accompanied by 3·5, 4·5, 5·5 "fast protons", respectively with further numerous additions of relatively slower protons. This agrees qualitatively with the large positive excess found by ROCHESTER and BUTLER[7]), and BUTLER, ROSSER and BARKER[8]), for smallish and medium sized showers.

4. Showers initiated by π-mesons.

We must expect that stars and meson showers can also be initiated by π-mesons. The process which is likely to give the most important contribution is the scattering of a π-meson by a nucleon. The direct capture is probably*) too small and the same is true for the conversion into a γ-ray and similar processes. When a meson of energy ε is scattered by a nucleon an energy E is transferred to the nucleon. E is determined within fairly narrow limits by the conservation laws, unless unreasonable assumptions about the angular distribution are made. When, for example, a meson of $\varepsilon = 5 \cdot 10^8$ e. v. is scattered by a nucleon the maximum energy transfer is $2{\cdot}3 \cdot 10^8$ e. v. and the average is of the order of $1 \cdot 10^8$ e. v.

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*) To our knowledge, calculations of the direct capture have only been made for non-relativistic mesons. For $\varepsilon \sim Mc^2$ one obtains a cross section for the capture of negative π's in Pb which is smaller, but not by an order of magnitude, than is required by the experiments quoted below. It is likely, however, that this decreases rapidly with increasing ε.

This gives rise to an evaporation star. The scattering of a meson will therefore, as a rule be accompanied by a star. When ε is of the order Mc^2, E rises rapidly, and when ε is larger than 10^9 e. v. E is of the same order as ε. For $\varepsilon \ll Mc^2$, E extends up to ε and on the average will be $\varepsilon/2$ or so. The nucleon receiving this large amount of energy will then, in the *same nucleus*, act in the same way as a primary nucleon does and create a penetrating shower including mesons. So when ε is very large, the meson will create a shower of penetrating particles which cannot, by appearance, be distinguished from a nucleon-initiated shower.

The cross section of a nucleus for shower production by a meson depends on the cross section for scattering by a nucleon. It is probable that this is smaller than the cross section for meson production by a nucleon. As a guidance we may use the cross section derived from the damping theory*).

This depends to some extend, within a factor 3 or so, on the form of meson theory used. For the Møller-Rosemfeld mixture the cross section was derived in ref.[10]) and the same method can be used for other varieties of meson theory. We give here, as an example the results for the charge-symmetrical pseudoscalar theory with pseudovector coupling (coupling constant f). The cross section depends essentially on the quantity

$$\tau = f^2 p^3/\varepsilon \tag{6}$$

where p is the momentum, ε the energy of the meson in units of the meson rest energy μc^2. The cross section, in units $(\hbar/\mu c)^2$, for the various processes is given in Table I, for $\varepsilon < Mc^2$.

Table I: $\varphi_{sc}/4\pi f^2 \frac{p}{\varepsilon}$

τ	0.5	1	2	3	4	8
$Y^+ N$	0.39	0.45	0.5	0.5	0.48	0.34
$Y^+ P$	0.45	0.72	0.79	0.7	0.6	0.35
$Y^+–Y^0$	0.2	0.35	0.26	0.15	0.09	0.09

$Y^+ N$ is the scattering of a positive meson by a neutron (or of Y^- by P), $Y^+ - Y^0$ the transformation of a pos. meson into a neutretto in a collision with a neutron. For small ε, φ_{sc} goes to zero

$$\varphi_{sc} = 4\pi f^4 \frac{p^4}{\varepsilon^2} \qquad (\tau \ll 1) \tag{6'}$$

*) Although in this theory a somewhat too drastic subtraction procedure was used no effective alternative has been found yet for meson processes.

(for $Y^+ - Y^0$ half this value). The asymptotic value for $\tau \gg 1$ is

$$\varphi_{sc} = 12\,\pi/\varepsilon^2 \qquad (\tau \gg 1\,,\ \varepsilon \ll Mc^2)\,. \tag{6''}$$

In the relativistic region $\varepsilon \gg Mc^2$ we have

$$\varphi_{sc} = \frac{16\,\pi}{\varepsilon\, M c^2} \qquad (\varepsilon \gg Mc^2,\ \tau \gg 1) \tag{6'''}$$

(ε = primary energy in rest system).

Now f^2 is not accurately known but is certainly of the order of magnitude 0·1 or 0·2. Allowing for a fairly large variation (0·05 — 0·5) we find for the avarage φ_{sc} for an energy of 10^9 e. v.

$$\varphi_{sc} = 0.4 - 0.9 \times \left(\frac{\hbar}{\mu c}\right)^2 = 0.7 - 1.6 \times 10^{-26}\ \text{cm}^2\,. \tag{7}$$

For $\varepsilon = 5 \cdot 10^8$ e. v. and $f^2 = 0{\cdot}2$ we get $\varphi_{sc} = 3 \cdot 10^{-26}$ cm^2.

It would be wrong now to add up the contributions of the individual nucleons because these overlap to some extend. The effect can easily be worked out. If d is the length of path through the nucleus, the probability for no collision to take place is $\exp(-N\varphi d)$, where φ is the cross section of an individual nucleon. Averageing over the geometrical nucleus we obtain for this probability

$$k = \frac{2}{b_A^2}\left[1 - (1 + b_A)\, e^{-b_A}\right], \qquad b_A = N \varphi\, d_A\,. \tag{8}$$

If, by $\varphi_A = \pi d_A^2/4$ we denote the geometrical cross section, the total cross section of the nucleus is

$$\Phi = \varphi_A\,(1 - k)\,. \tag{9}$$

If b_A is small, (9) goes over into $\Phi = \pi/6\; N d_A^3\, \varphi = A\varphi$, where A is the total number of nucleons, but for large b_A, Φ tends to a maximum value φ_A, as it must*).

For the values (7) of the cross section and Pb, b_A ranges from 0·9 to 2·1. Thus the total cross section of Pb for the scattering of a meson, by (8) and (9), should be

$$\Phi = 0.4 - 0.7\,\varphi_{\text{Pb}}\,, \qquad \text{for } \varepsilon = 10^9 \text{ e. v.} \tag{10}$$

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*) If $A\varphi > \varphi_A$, this means that more than one collision takes place in the same nucleus.

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The overlapping effect makes the result less sensitive to changes in the theory than is the individual cross section. For $\varepsilon = 5 \cdot 10^8$ e. v. and $f^2 = 0.2$ we get $\Phi = 0.8\ \varphi_{Pb}$. Pseudoscalar coupling ($f^2 = 0.2$) gives about $0.8\ \varphi_{Pb}$ at 10^9 e. v., and nearly $1 \cdot \varphi_{Pb}$ at $5 \cdot 10^8$ e. v.

It is therefore probable that for the meson energies which are mainly in question the cross section for nuclear interaction will be of the order of, say, half the geometrical cross section in heavy elements, at any rate between 0.4 and 0.9 φ_A. This is roughly in accord with the interaction length of shower particles determined by Butler, Rosser and Barker[8]). They found 30 cm Pb for this quantity, which, however, is still very inaccurate. The interaction length corresponding to φ_{Pb} is 12.5 cm Pb so that we get a cross section $\sim 0.4\ \varphi_{Pb}$, in agreement with the lower value (10).

We suggest therefore tentatively that the penetrating showers initiated by mesons are due to nuclear scattering. An indication for the fact that the interaction of π-mesons with nucleons is smaller than that of 2 nucleons at high energies can also be seen in the fact that fast π-mesons can escape from the nucleus whereas fast nucleons cannot.

Also here the possibility of multiple processes must be considered: A primary π-meson may split up into several mesons during the scattering. For this simplest type of multiple process the theory of damping leads to the result that the frequency of multiple processes to that of single scattering is less than 1:10. If the energy of the primary π-meson is very high, a decision of whether the process was single scattering or a multiple process is as difficult as for nucleon initiated showers. If the energy is about 10^9 e. v. a decision may be easier. Such a meson could split up into 2 or 3 secondary mesons, but if the single scattering is predominant, the energy transfer to the recoil nucleon will not be high enough to make the production of a further meson very probable.

Another interesting question that might be accessible to an experimental test is the transformation into a neutretto. In showers due to scattering, the secondary meson should be visible except when it is so slow that it can be captured in the same nucleus. If the meson is transformed into a neutretto no secondary meson is seen. Perturbation theory leads to the result that the transformation is half as frequent as the scattering but the theory of damping (Table I, large values of τ) shows that it decreases much more rapidly with ε than the scattering. With increased experimental material the point might be checked.

5. Angular distribution.

The general features of the angular distribution are a consequence of the conservation laws only. They permit us to draw conclusions about the energies involved but not about the mechanism of meson production. The finer details depend primarily on the angular distribution of the elementary process wheter this be a multiple or single process. In both cases one can easily invent a model to suit, within the general limits set by the conservation laws, any observation. A little more can be said in cases where a shower consists of 2 parts with essentially different angular distributions. As an example we consider the shower by KAPLON, PETERS and BRADT[9])*). The shower is initiated by an α-particle and consists of 56 particles, i. e. 14 particles per incident nucleon. Six of these are contained in a narrow bundle within a half angle of 3^0 the remainder ("wide shower") forms angles with the incident direction of 3^0—60^0, with most of the second group contained in the region 3^0—20^0. If the energy of the primary particle (per nucleon) is E_0 it follows from the conservation laws the angle of meson emission as well as the deflection of the primary is of the order

$$\sin \vartheta = \sqrt{\frac{2\,Mc^2}{E_0}}. \tag{11}$$

Putting $\vartheta = 3^0$, we get $E_0 \sim 10^{12}$ e. v. This must be some average of the energy of the primary (per nucleon) before entering and after leaving the nucleus, if the narrow shower is to be accounted for by the action of the primary. Now in each collision secondary nucleons are produced.

If we call "secondary" always the slower of the two nucleons it can again be seen from the conservation laws that the angle ϑ' formed by the secondary and the primary lies in the interval

$$\vartheta \leqslant \vartheta' \leqslant \vartheta_m, \qquad \sin \vartheta_m = 1 - \frac{\bar{\varepsilon}}{E_0} \tag{12}$$

where $\bar{\varepsilon}$ is the average energy of the meson produced**). We have assumed that $\bar{\varepsilon}/E_0 \sim 1/8$. Thus the recoil nucleons are contained

*) The shower has been described and discussed at length by the authors, assuming multiple production. The possibility that the same phenomenon can equally well be described in terms of pure plural production seems to have escaped the notice of the authors although they have come to the conclusion that it is difficult to explain the phenomen by one single multiple act.

**) This is true if the angular distribution of the meson is isotropic in the centre of mass system.

in the half-angle between 3^0 and 60^0 and they will produce mesons again within this half angle. The distribution of angles will be a more or less rapid decrease from the smallest angle (3^0) to a larger angle. The rate of this decrease depends on the model used and clearly any smooth distribution can be obtained by a suitable model. We refrain from inventing such a model, but it is clear that the wide shower can be accounted for by the action of the recoil nucleons. Moreover it is seen from Fig. 1 immediately that when the total number of mesons produced is 14 (per one incident nucleon) about 8 are primary mesons and 6 secondary, in approximate agreement with the number of particles found in the narrow and wide showers respectively. Actually the number of mesons found in this shower is somewhat smaller than one should expect for the exceedingly high energy of the primary. This suggests that the length of passage through the nucleus was rather less than the nuclear diameter. This makes it also understandable why other showers with the same number of shower particles (14) do not show this particular angular distribution. In a more or less diametrical passage through the nucleus a smaller primary energy suffices to produce 14 charged mesons, the order of magnitude is $E_0/E_c \sim 30$ and if $E_c = 10^9$ e. v., $E_0 \sim 3 \cdot 10^{10}$ e. v. The angular spread of this shower, even of the primary mesons, would be, (11), of the order of 20^0, with larger angles for the secondary mesons. This is the type of angular distribution frequently found for such showers.

Concluding remarks: In the foregoing we have stressed the plural aspects of shower production and have shown that all the conspicuous features of the penetrating showers can be understood. It is equally clear that the same features can be explained by assuming multiple production (as has been shown by many authors, although some authors have been forced to assume a mixed plural-multiple process). That such a dual interpretation of the events is possible is hardly surprising: Most of the conspicuous features of penetrating showers are mere consequences of the conservation laws. The shower size is limited above all by the total energy available. The limitation due to the finite size of the nucleus, if the process is plural, is of much lesser importance for usual shower sizes and heavy nuclei. Showers of 5—15 mesons say, owe their origin nearly all to primaries with energy just enough to produce them, and the contribution from primaries with much higher energies is almost negligible. Similarly, the angular distribution is largely determined by the conservation of momentum. It makes no difference whether a group of mesons is produced in one elementary act or in quick succession by the same energetic primary. Arguments, which have been put forward by many authors, trying to derive evidence for multiple processes from the broad features of penetrating showers, are therefore, in our opinion, misleading.

The finer features of a shower can only be explained by more specific assumptions about the energy and angular distribution of the elementary process. These cannot be derived yet reliably from any theory. The details of a shower are more affected by the assumptions one makes concerning the elementary process than they are dependent on whether or not meson production takes place in one elementary act or in quick succession.

Evidence for or against multiple processes must therefore await experiments in hydrogen (or at least light nuclei).

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