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# On a System of Fields
# Free of Divergences of the Mass-renormalization Type

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The procedure of the so-called "realistic" regularization has been applied with some success to the problem of the self-energy of the electron and to the problem of vacuum polarization. Pais[1]) and, independently, Sakata[2]) have shown that the self-energy of charged spinor particles may be removed by introducing an interaction with a neutral scalar meson (called briefly $C$-meson). It has also been shown[3])[4])[5])[6]) that the gauge invariance of the vacuum polarization current and the vanishing of the photon self-energy may be secured if the electromagnetic field is coupled with fermions as well as with bosons. Now, the question arises as to whether the divergences of the mass renormalization type may be removed from the whole system of fields i. e. whether the self-energy of a $C$-meson itself due to the coupling with charged fermions will be compensated by a suitable interaction with charged bosons and whether the self-energies of charged bosons due to the interaction with photons and with the $C$-mesons will compensate each other without any further assumption of additional fields. The answer to this question is affirmative (at least in the $e^2$-approximation).

We introduce the following interaction energy density:

$$H(x) = \sum (H_F + H_B + H_{FC} + H_{BC}) \qquad (1)$$

where

$$H_F(x) = -j_{\mu F}(x)\, A_\mu(x)$$

is the usual interaction between fermions and the electromagnetic field, $H_B$ means the usual interaction between scalar bosons and the electromagnetic field (see e. g. reference no.[5]) while

$$H_{FC} = g \bar{\psi}_F \psi_F \Phi_C \quad \text{and} \quad H_{BC} = f \varphi_B^* \varphi_B \Phi_C$$

mean the interaction of fermions or bosons with the $C$-meson. The sum in (1) is extended over the number of fields of each type.

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The computations of the self-energies in question have been performed by standard methods developed by SCHWINGER[7]). We represent each self-energy term in form of integrals over products of the invariant $\Delta$ and $\Delta^{(1)}$ functions and their derivatives, use the well known Fourier representations of these functions and introduce invariant parameters $u$, $v$ by means of the identities:

$$\delta(A) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} du\, e^{iAu} \tag{2}$$

$$\frac{1}{B}\left(\delta(A+B) - \delta(A-B)\right) = \int_{-1}^{1} dv\, \delta'(A+Bv). \tag{3}$$

This procedure enables to put all the infinities in a form of logarithmically divergent integrals over the invariant parameters. Let us take for example

$$\Delta^{(1)}(0) = \frac{1}{(2\pi)^3} \int d^4k\, \delta(k_\mu^2 + \varkappa^2).$$

By integrating this by elementary methods we would get a quadratically divergent integral:

$$\int_0^\infty dk \frac{k^2}{\sqrt{k^2+\varkappa^2}}$$

but using (2) together with the well known formula:

$$\int d^4k\, e^{i k_\mu^2 u} = \frac{i\pi^2}{u^2} \operatorname{sign} u$$

we obtain

$$\Delta^{(1)}(0) = \frac{i}{(4\pi)^2} \int_{-\infty}^{+\infty} du \frac{e^{i\varkappa^2 u}}{u^2} \cdot \operatorname{sign} u$$

which is only logarithmically divergent. In the special case of $\varkappa = 0$ this last integral becomes zero on symmetry grounds (compare reference no. [8]).

The square of the self mass of charged scalar (or pseudoscalar) bosons due to the interaction with the $C$-meson is:

$$\delta m_B^2 = \frac{f^2}{(4\pi)^2}\left(\frac{m_B^2}{m_C^2} - 1\right) \int_0^\infty du \frac{\cos u}{u} + \text{finite terms} \tag{4}$$

while the same quantity due to the interaction with the electromagnetic field is:

$$\delta m_B^2 = \frac{3}{(4\pi)^2} e^2 m_B^2 \int_0^\infty \frac{\cos u}{u} + \text{finite terms}. \tag{4'}$$

Both infinities may be brought to cancellation if

$$m_B < m_C.$$

The squared self-mass of the $C$-meson due to the interaction with charged Bose field is:

$$\delta m_C^2 = -\frac{f^2}{2(4\pi)^2} \int_0^\infty du \frac{\cos u}{u} + \text{finite terms} \tag{5}$$

while the analogous quantity due to the coupling with a spinor field is:

$$\delta m_C^2 = \frac{g^2}{(4\pi)^2} (6 m_F^2 - m_C^2) \int_0^\infty du \frac{\cos u}{u} + \text{finite terms}. \tag{5'}$$

The last two infinities may be brought to cancellation provided

$$6 m_F^2 > m_C^2.$$

If we introduce the condition for the finiteness of the electron self-energy[1])

$$g^2 = 2 e^2$$

and the two conditions for the gauge invariance of the vacuum polarization current[4]):

$$N_B = 2 N_F$$

$$\sum_1^{N_B} m_B^2 = 2 \sum_1^{N_F} m_F^2$$

then we easily see that all the conditions for compensation are compatible. The mathematically simplest way to satisfy all the conditions of finiteness is to assume a single spinor field (e. g. electrons whose mass constant is denoted simply as $m$) and two charged scalar fields with the same masses. In this case we need only one $C$-meson field and the values of the mass and coupling constants are:

$$f = 2em \quad \text{or} \quad f = 3em$$

$$m_c = 2m \quad \text{or} \quad m_c = \sqrt{3/2}\, m$$

$$m_B = m.$$

The above system, distinguished by its mathematical simplicity, is difficult to accept from the physical point of view: all the particles in question possess masses of the order of magnitude of the electron mass and, thus, do not deserve the name of mesons. But more general solutions exist too if we introduce (besides electrons) another spinor field with a large mass constant.

Concluding we state that in frame of the relativistically invariant formulation of quantum field theory exist comparatively simple systems of fields free of divergences of the mass renormalization type (at least in the $e^2$-approximation). Quantum electrodynamics constitutes the main part of such closed systems of mutually compensating fields.

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