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Stationary states of light nuclei

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1. The interaction between two nucleons.

The present, very imperfect treatment of nuclear systems is based on the assumption that the total interaction between the constituent nucleons is primarily due to the interactions between pairs of nucleons, while many-body interactions would only contribute to higher approximations with respect to the nucleon velocities. Whether this assumption is quite justified remains open to question, since, as we shall see, theoretical calculations concerning systems of 3 or 4 nucleons, based on pair interactions only, exhibit large discrepancies from empirical results. A quantitative estimate of many-body interactions is at present precluded by the fundamental difficulties of field theory; we must therefore leave this question in a most unsatisfactory state, and base the following discussion on the assumption of pair interactions.

Our first task will then be to establish the most probable expression for the interaction operator between two nucleons; recent investigations make a renewed survey of this problem desirable. Let us start from the consideration of the interaction between neutron and proton, as revealed, on the one hand, by the experiments on scattering of very fast neutrons by protons, and on the other, by the properties of the ground state of the deuteron. The best theoretical approach to the first problem, in the present state of field theory, is the relativistic generalisation of Born's method, due to Moller. This method can readily be applied to the problem of proton-neutron scattering by assuming that the interaction between the particles results from a coupling of these particles to a meson field. It is then possible to derive in a consistent way the contributions to the scattering cross-section, not only from the static interaction usually considered, but also from the additional interaction terms of the first and second order in the nucleon velocities.

An expression for the total scattering cross-section has recently been obtained by Marty*), using Moller's method and assuming

*) Not yet published.

a coupling of the nucleons with a symmetrical mixture of pseudoscalar and vector meson fields, with arbitrary coupling constants. The vector field involves two constants g_1, g_2 corresponding to vector and tensor source densities, respectively; the pseudoscalar field is represented, owing to the elastic character of the scattering, by a single combination*)

$$f_3 \equiv f_2 + f_1 \frac{M_m}{2\,M}$$

of the two analogous constants f_1, f_2 (corresponding to pseudoscalar and pseudovector source densities), in conformity with Dyson's transformation. The dependence of the total cross-section on the coupling constants can be described as follows: There appears, in the first place, the expression which one would have obtained on Born's approximation, i. e. from the static interaction only, multiplied by the factor $[1 - (p/M)^2]$ (p denoting the momentum of the particles in the barycentric system of reference); to this term only the vector meson interaction contributes, and the combination of the coupling constants g_1, g_2 occurring in it are simple polynomials of the 4th degree. Besides this term, there is a further contribution in (p^2/M^2) whose coefficients are similar polynomials, but involving also the constants g_2 and f_3 multiplied by the large parameter (M/M_m). For an estimation of the relative importance of the two parts of the expression for the scattering cross-section, it is natural to take for the coupling constants g_1, g_2, f_3 values of the order of magnitude indicated by an analysis of the ground state of the deuteron based on the assumption of static interaction only. It then appears that for a value of the momentum p corresponding to incident neutrons of 90 MeV energy, the second part (in spite of the occurrence of the large coefficients M/M_m) contributes a correction of the order of a few percent of the first, which tends to increase the value of the cross-section corresponding to the assumption of static interaction, and thus to bring it farther from the experimental result. The discussion of the differential cross-section has not yet been completed: it remains to be seen whether, within the scope of symmetrical meson theory, there is still sufficient latitude in the choice of the precise values of the coupling constants to achieve agreement with the observed angular distribution of the scattering particles, without coming in conflict with the properties of the deuteron.

This leads us to a revision of the deuteron problem: does the usual treatment based on the assumption of a static interaction form an

*) The masses of a nucleon and a π-meson are denoted by M and M_m, respectively.

adequate approximation, or are velocity-dependent couplings, hitherto not properly considered, in fact of paramount importance? As is well known, the issue is obscured by the strongly singular character of the static interaction, which forces us, so long as we treat it separately, to introduce into the problem an arbitrary element in the form, e. g., of a cut-off of the static potential at small distances. One may expect that a further step in the approximation, similar to that which led to the relativistic expression for the scattering cross-section, discussed above, will give a simple answer to the question. The investigation of the ground state of the deuteron from this point of view has only recently been taken up, however, and its result will be awaited with great interest.

In the meantime, we may perhaps derive some guidance from a discussion of the problem in the static approximation, the arbitrary cut-off involved in this treatment being regarded as a rough way of accounting for the effect of the higher approximations. For the most general combination of meson fields, the effective static potential, for the ground state of the deuteron, has the form

$$-\mathfrak{J}\left[1+\alpha\,\mathfrak{D}^{(12)}\left(1+\frac{3}{\varkappa r}+\frac{3}{\varkappa^2 r^2}\right)\right]\frac{e^{-\varkappa r}}{r},$$

where $\mathfrak{J}$ and α are certain combinations of the coupling constants of the meson fields to the nucleons, while $\varkappa \equiv M_m c/h$ and

$$\mathfrak{D}^{(12)} \equiv (\vec{\sigma}^{(1)}\vec{x}_0)(\vec{\sigma}^{(2)}\vec{x}_0)-\frac{1}{3}(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})$$

is the well-known operator of "axial dipole coupling" between the spins $\vec{\sigma}^{(i)}$ of the nucleons, depending on their orientation with respect to the line joining them (represented by the unit vector $\vec{x}_0$). For a given value of $\varkappa$, the wave-equations corresponding to the above potential, cut-off in a definite way for distances smaller than some critical value r_c, can be solved numerically for various sets of values of the parameters, $\mathfrak{J}$, α, r_c. Each solution corresponds to definite values for the binding energy, the quadrupole moment and the admixture of ^3D-state (derived from the magnetic moment): the adjustment to the empirical values of these quantities yields, in principle, the values of the parameters defining the effective potential. Extensive calculations of this type have been carried out by GROSJEAN*), first for a meson mass of 225 m, and recently also for the higher value $M_m = 285\ m$ of the mass of the π-meson res-

*) Not yet published.

ponsible for the nuclear field. His results present in both cases the same general features. In the first place, the value of α must be about $^1/_2$; this does not correspond to any single type of meson field and would thus point to the necesssity of assuming some mixture of such fields: we shall adopt, as the simplest one, a mixture of pseudoscalar and vector fields. For values of α in the neighbourhood of $^1/_2$, it then appears that the quadrupole moment is quite insensitive to the value of the cut-off radius, but that the admixture of D-state and the strength $\mathfrak{J}$ of the central part of the potential both vary almost linearly with r_c. Since the admixture of D-state is fixed only with poor accuracy (owing to uncertain relativistic corrections to the magnetic moment), neither $\mathfrak{J}$ nor r_c are accurately determined by this method. However, the possible $\mathfrak{J}$-values are of the same order of magnitude as the strength of the effective potential for the ^1S state, as estimated from the cross-section for scattering of very slow neutrons by protons. This means that the coupling constants g_2^2/hc, f_3^2/hc are of the order of magnitude 10^{-1}, and that their difference, as indicated by the rather large value of α, is of the same order. The constant g^2/hc, on the other hand, is still allowed a wide range of variation from zero to values only somewhat smaller than those of the other constants. It must be stressed that the high value of the cut-off radius, which is given by $\varkappa r_c \approx 0{,}7$, rather weakens the reliability of these results. Nevertheless, one might perhaps conclude that *the interaction between neutron and proton can be accounted for by a mixture of pseudoscalar and vector meson fields, in such proportions that the binding of the deuteron is largely due to the resulting axial dipole potential.* For the treatment of effects involving only small energies (up to 20 MeV, say), the model of nuclear potential proposed by Rarita and Schwinger (in which both the central and the dipole forces are represented by potential wells of the same width and appropriate depths) provides a convenient schematization. But it cannot, of course, account for the proper field effects already prominent in the domain of energies about 100 MeV.

In the preceding discussion, only one meson mass has been assumed. Attempts at introducing mesons of different masses in the nuclear field, according to Schwinger's proposal, have not proved successful. Empirical data on slow neutron diffraction, on the other hand, when analysed by means of the schematic potential well model, show that different widths must be assumed for the wells corresponding to the effective potentials for the ^3S and ^1S states. This conclusion, however, depends sensitively on the form of potential adopted: it appears*) that the empirical results can be explained on the basis of meson potentials of single range for both types of states (as was assumed in the above discussion). In this connexion, it must be pointed out that the range value derived from the slow neutron diffraction and scattering data for the ^1S potential, in contrast to previous statements to the contrary, appears to be quite compatible with the range of the ^1S potential between two protons derived from proton-proton scattering experiments. There is thus no reason so far to doubt the (approximative) "charge independence" property of the nuclear potential, at any rate for ^1S states. Moreover, the scanty indications on the effective potential for ^3P states derived from proton-proton scattering data at higher energies (10... 14 MeV) are not incompatible with the extension of the charge independence property to all types of stationary states. The general conclusion of the preceding discussion of the evidence from two-nucleon systems may be condensed in the following expression for the interaction potential between two nucleons *of low velocity*:

$$\frac{1}{3}\,\underline{\tau}^{(1)}\,\underline{\tau}^{(2)}\,\mathfrak{J}\left[\gamma + (1-\gamma)\,\vec{\sigma}^{(1)}\,\vec{\sigma}^{(2)} + \alpha\,\mathfrak{D}^{(12)}\left(1 + \frac{3}{\varkappa r} + \frac{3}{\varkappa^2 r^2}\right)\right]\frac{e^{-\varkappa r}}{r},$$

with due emphasis on the many uncertainties which still beset the argument.

This formula, however, is still incomplete in one respect, about which two-nucleon systems cannot yield any evidence. Besides the axial dipole coupling, another type of non-central coupling is possible on general invariance grounds, namely, a spin-orbit coupling of the type

$$\mathfrak{M}^{(12)} \equiv (\vec{\sigma}^{(1)} + \vec{\sigma}^{(2)}) \cdot (\vec{l}/\hbar),$$

where $\vec{l}$ denotes the orbital angular momentum with respect to the centre of gravity of the nucleon pair. Such a coupling does not give any contribution to S-states of a two-nucleon system, but it might play a part in the stationary states of more complex systems. A spin-orbit coupling of relativistic origin (of the second order in nucleon velocities) must be present in any case; roughly, it will have the form, on meson theory,

$$\sim \frac{1}{3}\,\underline{\tau}^{(1)}\,\underline{\tau}^{(2)}\,\mathfrak{J}\left(\frac{M_m}{M}\right)^2 \mathfrak{M}^{(12)}\left(\frac{1}{\varkappa r} + \frac{1}{\varkappa^2 r^2}\right)\frac{e^{-\varkappa r}}{r}.$$

A vector meson field will in general also give rise to a coupling of

*) See J. Blatt and J. Jackson, Phys. Rev. **76**, 18, (1949); H. Bethe, Phys. Rev. **76**, 38 (1949).

the same form, but with a factor $g_1\, g_2\, (M_m/M)$ instead of the factor $^1/_3\, \mathfrak{J}\, (M_m/M)^2$: if the constants g_1, g_2 are of the same order of magnitude, this effect is of the first order in the nucleon velocities. In view of the evidence, discussed in the following, which more or less directly points to the existence of considerable spin-orbit couplings in nuclei, this theoretical possibility should certainly be kept in mind.

2. The ground state of ^3H.

In the deuteron, the non-central interaction of the axial dipole type brings about a breakdown of the conservation of orbital angular momentum, with the result that the ground state, which belongs to the triplet system, is a mixture of S and D state. If the system contains more than two nucleons, there is even no conservation of spin; in the case of ^3H, the doublet and quartet systems are also combined. The ground state of ^3H, whose total angular momentum is $^1/_2$, will therefore be a combination of the form $^2S_{\frac{1}{2}} + {}^2P_{\frac{1}{2}} + {}^4P_{\frac{1}{2}} + {}^4D_{\frac{1}{2}}$. Moreover, in this mixture there are two types of doublet (S and P) states, according as the wave-function is symmetrical ($\bar{S}$, P) or antisymmetrical ($\tilde{S}$, $\tilde{P}$) in the spins of the neutrons; likewise, there are two types of ^4D states (D_s, D_a), distinguished by the symmetrical or antisymmetrical character of the dependence of the wave-function on the variables $\vec{x}$ (radius vector joining the neutrons) and $\vec{X}$ (radius vector joining the proton to the centre of gravity of the neutrons). To a first approximation, one may assume that the two neutrons will be "paired" in a configuration with saturated spins, corresponding to a $^2\tilde{S}_{\frac{1}{2}}$ state. By axial dipole interaction, this state will be directly coupled only to the $^4D_{\frac{1}{2}}$ state; any admixture of other states in appreciable amount will be an indication of the occurrence of other couplings, either of the spin-orbit or of the 3-body type.

Definite indications of this kind can in fact be inferred from two fundamental properties of the triton: its binding energy and its magnetic moment. A calculation of the binding energy on the assumption of a Rarita-Schwinger interaction between the pairs of nucleons (i. e. involving only axial dipole coupling) yields a much smaller absolute value than the empirical one: it appears that the axial dipole coupling, so efficient in bringing about the deuteron binding, has a much smaller total effect in the 3-nucleon case. The fact that the magnetic moment is *larger* than the proton moment likewise cannot be understood on the basis of axial dipole coupling:

for a magnetic moment equal to the proton moment (as would correspond to a pure $^2\tilde{S}_{\frac{1}{2}}$ state) could only be lowered by an admixture of ^4D_s state. It is true that a nuclear field effect, the "exchange moment", might give an additional contribution in the right direction, but this could hardly be sufficient to bring the needed overcompensation of the ^4D_s term. On the other hand, a more complicated mixture of states would lead to an additional magnetic moment which may be of either sign. Whether agreement with experiment in both cases could actually be reached by taking into account spin-orbit or 3-body interactions has, however, not yet been investigated.

3. The quasi-atomic model.

Experimental data concerning the stationary states of light nuclei are rapidly accumulating. The *energies and widths of excited levels* can be inferred from the occurrence of resonances in the yield of nuclear reactions induced by impact of particles or γ-rays, or from the study of β-decay processes, especially when such processes are accompanied by the emission of γ-rays or electron pairs. Methods of high precision have been developed to measure the *total angular momenta, magnetic moments and quadrupole moments of the ground states* of stable and even some unstable nuclei. In favourable cases, the application of selection rules also allows inferences to be made concerning the *quantum numbers and parity of excited states*. Another source of information bearing more especially on the *orbital quantum numbers* entering into the composition of excited states is provided by the study of the angular distribution of particles ejected in the final stage of the nuclear reaction studied.

The theoretical approach to the problem of the stationary states of nuclear systems, on the other hand, is hampered by the lack of adequate methods of approximation for treating assemblies of closely coupled particles. The general view-points of group theory offer only a rather loose frame: the classification of nuclear states into *supermultiplets*, characterized, for a given mass number A, by 3 quantum numbers (P, P', P'') with the following interpretation. Consider the operators

$$\underline{T}_3 = \frac{1}{2} \sum_i \tau_{\underline{3}}^{(i)}, \qquad S_z = \frac{1}{2} \sum_i \sigma_z^{(i)}, \qquad \underline{Y}_z = \frac{1}{2} \sum_i \tau_{\underline{3}}^{(i)} \sigma_z^{(i)},$$

which represent, respectively, half the neutron excess, the component of the total spin in an arbitrary direction, and the component of the difference between total neutron and proton spin in this direction. Then P is the largest eigenvalue that any of these operators can take in the supermultiplet, P' is the largest eigenvalue of a second one among these operators which is compatible with the value P of the first, P'' is the largest eigenvalue of the third compatible with the values P, P' of the other two.

Pure supermultiplet states (characterized by eigenvalues of T_3, S_z, $\underline{Y}_z$) could only occur if the nuclear interactions were central forces independent of the spin and isotopic variables of the constituent nucleons; in general, they would be degenerate, being superpositions of states belonging to different spin or charge multiplicities. Central forces depending on spin and isotopic variables will split up the supermultiplet states into different spin and charge multiplets, but in this splitting, supermultiplet states with different values of $\underline{Y}_z$ will in general combine, so that $\underline{Y}_z$ ceases to furnish a quantum number. Non-central couplings will further mix the multiplet substates; the resulting energy spectrum could only be obtained by the consideration of a more specific nuclear model. The Coulomb repulsion between the protons has the effect of decreasing the (absolute) binding energy values of supermultiplet states with decreasing neutron excess; for light nuclei, however, this is a rather unimportant feature.

The supermultiplet scheme is useful for the establishment of general regularities of a semi-qualitative nature (such as selection rules for β-decay, peculiarities of the mass-defect curve, distribution of magnetic moments), but, of course, it hardly provides an adequate starting point for actual calculations of energy levels in specific cases. For this purpose, recourse must be had to some model to which the methods of quantum mechanics can be applied. The *quasi-atomic model* consists in assuming that in first approximation each constituent nucleon occupies an individual state independently of the others, and in applying to the interaction between the nucleons the usual methods of approximation developed for the treatment of atomic systems. In the latter case, this procedure is justified by the predominance of the electrostatic field of the atomic nucleus; in the case of nuclear systems, however, one has to introduce for the definition of the individual states of the nucleons some fictitious "average nuclear field" which, on account of the saturation properties and above all of the non-additive character of the nuclear forces resulting from the existence of considerable many-body interactions, does not correspond even roughly to any physical reality. The greatest shortcoming of the calculations hitherto carried out by this method, however, is that they use only central interactions. It is above all the neglect of non-central forces which makes the results of such calculations quite unreliable: even the order of succession of the multiplet levels cannot be predicted with any certainty.

On the quasi-atomic model, the individual nucleon states may be ascribed orbital quantum numbers, and the building up of the ground states of nuclei of increasing mass number may accordingly be pictured as a gradual filling up of the "shells" corresponding to the successive individual nucleon states. Any nuclear potential accounting for the properties of two-nucleon systems will favour a configuration of any pair of constituent nucleons of the same shell with an *even* value of the total orbital momentum: for it will lead to an attraction between the two nucleons in such a configuration, while (on the charge independence hypothesis) we shall expect a repulsive interaction in all configurations of odd total orbital momentum. More specifically, in an even configuration, the attraction between two nucleons with opposite spins will in any case be purely central, while the interaction between a proton and a neutron with parallel spins will be a stronger attraction, which might be central or non-central. These forces will in the first place tend to the formation, within each shell, of groups consisting of a neutron pair and a proton pair, both with saturated spins ("α-clusters"). Any two additional like nucleons, in the shell, will tend to "pair" themselves in an even singlet configuration: this general conclusion, however, does not suffice to give an interpretation of the fact that even nuclei have zero angular momentum and no magnetic moment, corresponding to a ^1S ground state.

When we try to apply the model to light odd nuclei, in which non-central forces become prominent, we run at once into still greater difficulties. An example of typical interest is that of the nucleus ^{10}B, whose ground state has recently been found to have a total angular momentum $J = 3$, quite at variance with expectation from the quasi-atomic model with central interactions only. In fact, on this model, the configurations of lowest energy of the nucleus ^{10}B consist of a filled s-shell (α-cluster) and a p-shell containing an α-cluster and a proton-neutron pair with parallel spins. On the assumption of central interactions, such configurations would give rise, in first approximation, to the states ^3S, ^3D, ^3D, ^3F, ^3G in order of decreasing binding: the ground state would thus be expected to have the angular momentum $J = 1$. Higher approximations will produce some displacement of these levels by mixture with higher levels of the same type: but this effect has been found by VAN WIERINGEN*) to be negligible. It is clear that in order to explain the presence of any level of higher orbital momentum below the ^3S state, non-central (and possibly many-body) forces must be assumed not merely to give rise to perturbations of levels corresponding to central interactions, but even to play a prominent part in the binding of the nucleus.

A more precise indication of the nature of the ground state of ^{10}B may be derived from the knowledge of its magnetic moment. In fact, the ground state of angular momentum $J = 3$ may be a mixture of the states ^3D, ^3F, ^3G mentioned above, with in addition the state ^1F from the singlet system belonging to the same supermultiplet. If the proportions of these states in the mixture are denoted by the symbols of the states between brackets, the magnetic moment may be written in the form

$$\mu = \mu_p + \mu_n + 1 - \left(\mu_p + \mu_n - \frac{1}{2}\right)\left[(^1F) + \frac{3}{4}\,(^3F) + \frac{7}{4}\,(^3G)\right];$$

in this easily derived formula, μ_p and μ_n denote the proton and the neutron moment, respectively, and (^3D) has been replaced by $1 - [(^1F) + (^3F) + (^3G)]$. From the experimental value $\mu = 1{,}794$ one therefore concludes that the ground state is mainly a ^3D state, but with an admixture of at least 22% of other states.

The discovery of the high angular momentum of ^{10}B has provided the solution of the riddle presented by the highly forbidden character of the β-transition $^{10}\mathrm{Be} \rightarrow {}^{10}\mathrm{B}$, in which the even isobar ^{10}Be must be expected to have zero angular momentum. Moreover, it has recently been found that the β-decay of the conjugated**) isobar ^{10}C into ^{10}B, which is an allowed transition, is accompanied by a γ-ray, so that it involves an excited state of ^{10}B which may well have an angular momentum 1. The quantitative relations between the isobars of mass 10, shown on the accompanying diagram, allow an interesting comparison of the nuclear structures involved***). The configuration of the conjugated even isobars ^{10}Be and ^{10}C can be pictured as one in which all pairs of like nuclei have saturated spins. Compare with these the configuration ^{10}B* of the odd isobar ^{10}B in which the proton and the neutron in excess of α-clusters have opposite spins: this will, of course, correspond to some excited state of the ^1S type. The differences of Coulomb energies between these

*) Not yet published.

**) Two nuclei are called *conjugated* if the one goes over into the other by changing protons into neutrons and vice-versa.

***) R. SHERR, H. MUETHER, M. WHITE, Phys. Rev. **75**, 282 (1949).

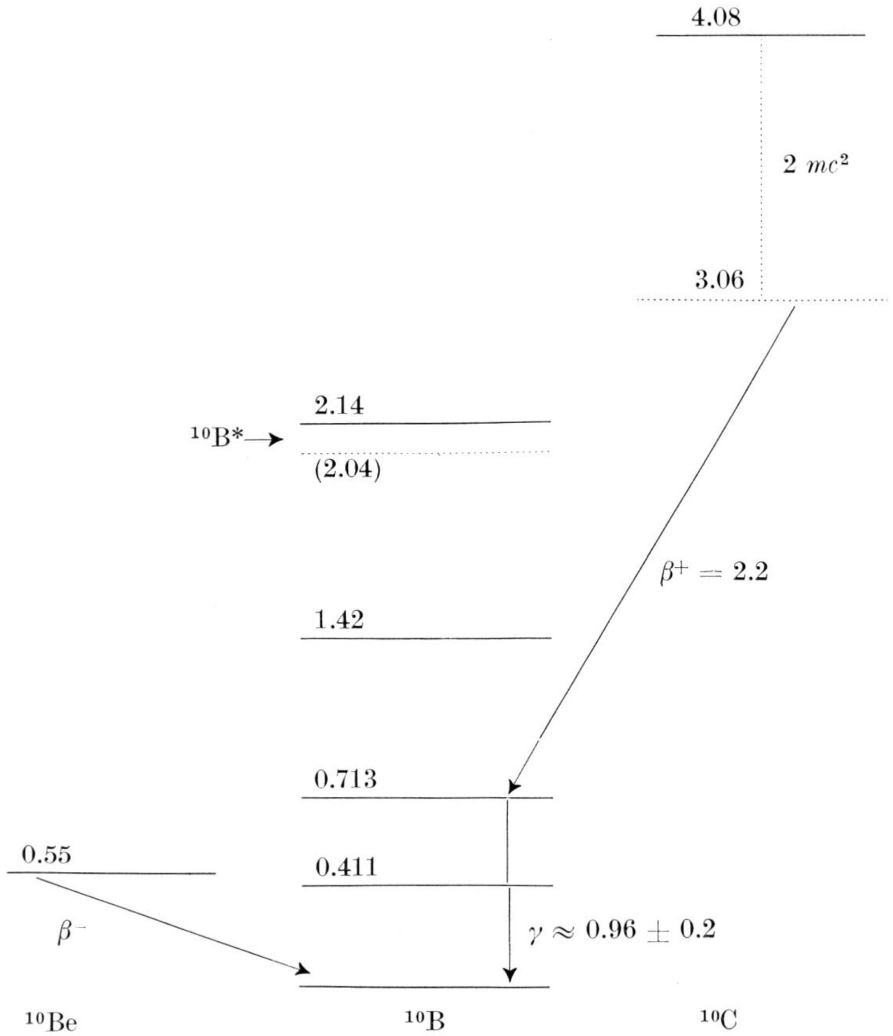
configurations (assuming the nuclear radii to be the same) are easily estimated at

$$^{10}\mathrm{C} - {}^{10}\mathrm{B}^* = 2{,}04\ \mathrm{MeV}, \quad {}^{10}\mathrm{B}^* - {}^{10}\mathrm{Be} = 1{,}48\ \mathrm{MeV},$$

whence

$$^{10}\mathrm{C} - {}^{10}\mathrm{Be} = 3{,}52\ \mathrm{MeV},$$

in remarkable agreement with the empirical result. We are therefore entitled to conclude that in the three isobaric configurations considered, the proper nuclear energy is the same: this entails not



The isobars of mass 10. (Energies in MeV)

only symmetry of nuclear interactions with respect to charge, but also charge independence of these interactions in configurations of the ^1S type.

4. The α-particle model.

At first sight, it would seem that a much better approach to the problem of nuclear states would be provided by the *α-particle model*, in which the nucleus is assumed to consist of α-clusters with the necessary number of additional nucleons. In the first place, a tendency to the formation of such α-clusters is, as we have seen, an obvious consequence of the properties of nuclear interactions. In fact, the binding energy of an α-particle is so large that if this energy is, on the average, assigned to each of the constituent α-clusters, the greatest part of the total binding energy of the nucleus is accounted for; the mutual binding of the α-clusters and additional nucleons is comparatively small. It would seem, therefore, that a model in which this large contribution to the binding energy is included from the start represents a considerable improvement on the quasi-atomic model, which is quite inadequate in this respect.

Unfortunately, on closer examination, the picture loses its treacherous simplicity. It cannot be supposed, of course, that the α-clusters retain their identity within the nucleus. One has rather to imagine that they form and dissolve continually in the course of chance encounters of nucleons in their chaotic motion. By studying some simple system, like ^8Be, and trying to represent it as a superposition of an 8-nucleon quasi-atomic system and a system of two α-particles, it is possible to obtain a rough estimate of the "degree of dissociation" of the α-clusters with respect to the "gas" of nucleons. The result is that this degree of dissociation is rather high, and this at once forces us to question seriously the accuracy of any "geometrical" model in which the assembly of α-clusters is compared to a close packing of rigid spheres held together by a certain number of "bonds" (determined by the number of points of contact of the spheres in the given configuration). Such doubts increase on closer inquiry into the nature of the "bond" between two α-particles: it appears that this bond is not primarily an ordinary interaction similar to the van der Waals intermolecular forces, but rather an interaction of the "exchange" type, conditioned by the exchange of nucleons between the clusters, and more nearly comparable to the chemical binding forces. In particular, the non-additive character of exchange interactions precludes any simple interpretation of the remarkable empirical fact that the binding energies of the light "α-nuclei" (i. e. nuclei consisting only of α-clusters) are proportional to the numbers of bonds of the corresponding geometrical models.

We must therefore abandon the hope of finding in the α-particle model a suitable starting point for the computation of nuclear binding energies. For more qualitative purposes, however, the model has many advantages over the quasi-atomic picture. It can be shown, for instance, that in spite of their transient character, the α-clusters may be expected to retain their cohesion during times long compared with the periods of vibration and rotation of the configurations in which they arrange themselves in the lightest α-nuclei. On this basis, it is possible to give a qualitative and even semi-quantitative description of the excited levels of these nuclei in analogy with the treatment of vibration and rotation levels of polyatomic molecules. An interesting feature of this treatment arises from the identity of the constituent α-clusters, which has the effect of pushing upwards a certain number of rotation levels according to the symmetry or quasi-symmetry of the configuration, thus explaining the absence of any rotational fine structure of low excitation energy, especially in heavier nuclei. A simple case of this kind is that of ^{8}Be, in which only the even values of the angular momentum are allowed by symmetry.

Some general regularities of magnetic moments and electric quadrupole moments of nuclei can be better understood, at least qualitatively, on the α-particle model than on any model in which the nucleons are considered individually. On the one hand, the values of the magnetic moments of odd mass nuclei can be roughly accounted for as if they were due to the individual motion of the odd nucleon in the field of the residual nuclear structure; on the other hand, the positive values of many quadrupole moments likewise point to the existence of a structure of some "rigidity" within the nucleus; closer examination shows that the picture offered by the α-particle model would just provide such a structure with suitable properties.

The α-particle model affords also a convenient starting point for a semi-quantitative treatment of the ground states of light nuclei differing from α-nuclei by an additional or a "missing" neutron. The extra particle or "hole" can be considered as moving in the field of the α-cluster configuration representing the α-nucleus, and the energy of its binding to this nucleus can readily be expressed in terms of the interaction energy operator between neutron and α-particle and the corresponding eigenfunctions of a neutron in the field of an α-particle. As an illustration of the method, let us consider the ^{8}Be configuration and the two neighbouring nuclei ^{7}Be and ^{9}Be. If ψ_1, ψ_2 represent the p-state eigenfunction of lowest energy of a neutron in the field of the α-particle 1 or 2, respectively, and if we assume this eigenfunction to be very concentrated around the centre of the α-particle, with a node at the centre, the eigenfunction ψ of a neutron or hole in the field of the two α-particles has either of the approximative forms

$$\frac{1}{\sqrt{2}}(\psi_1 + \psi_2) \qquad \text{or} \qquad \frac{1}{\sqrt{2}}(\psi_1 - \psi_2).$$

In order to obtain the lowest energy state of the corresponding nucleus, we must choose the eigenfunction of lowest energy in the case of an additional neutron, of highest energy in the case of a hole. These two cases correspond to the first and second of the above eigenfunctions, respectively, as appears from a consideration of the numbers of nodes of these functions. Now, let V_1, V_2 be the interaction energy operators of the α-particles 1,2 with the neutron, K the kinetic energy operator of the neutron, $H = V_1 + V_2 + K$ its Hamiltonian. Define the energy of the neutron in the field of an α-particle

$$B \equiv \int \psi_1^* (V_1 + K)\, \psi_1,$$

the average interaction energy of a neutron bound to one of the α-particles with the other

$$R \equiv \int \psi_1^* \, V_2 \, \psi_1,$$

and the exchange interaction energy

$$Q \equiv \Re \int \psi_1^* \, H \, \psi_2.$$

Then the energy of the additional neutron, i. e. the difference between the energies of ^{9}Be and ^{8}Be $+ n$, is approximately given by $B + R + Q$; and the difference between the energies of ^{7}Be and ^{8}Be $- n$ is given by $B + R - Q$. A complete discussion of this method, including a comparison with the empirical data, will be found in HAFSTAD and TELLER's fundamental paper on the α-particle model*). An extension of the same procedure to the case of two additional nucleons, and in particular to the nucleus ^{10}B, might perhaps reduce to manageable proportions the problem of the interaction between the two additional particles in this nucleus, on which the explanation of the observed value of the angular momentum ultimately depends.

*) L. HAFSTAD and E. TELLER, Phys. Rev. **54**, 681 (1938).