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ℓ ist ein Punktnummerindex, über den die Punkte im Gesamtmusterverband identifiziert werden können. Die Bedingungen für die Lagekorrespondenzen eines transformierten Punktes (ℓ) lauten

$$\bar{x}_{\ell(j-m)} = \dots = \bar{x}_{\ell j} = \bar{x}_{\ell(j+n)}, \quad (11a)$$

$$\bar{y}_{\ell(j-m)} = \dots = \bar{y}_{\ell j} = \bar{y}_{\ell(j+n)}.$$

Die Summe $m+n+1$ repräsentiert die Anzahl der zu einem bestimmten Punkt gehörenden Muster (z. B. $m+n+1=4$ für jeden Punkt $P_1, \dots, P_4$ aus der Abbildung).

Das Gleichungssystem zur Grauwertkorrelation lautet hier

$$-e_j = A_j'x_j' + A_j''x_j'' - \ell_j; P_j \quad (12)$$

$j = 1, \dots, p; p \dots$ Anzahl der Einzelmuster im Verband

Dieser Ansatz (12) ist formal identisch mit dem aus (1a), jedoch markiert hier der Index j das Einzelmuster im Verband.

Fassen wir die transformierten Punktkoordinaten als beobachtete Größen auf, so können wir die Korrespondenzbedingungen der identischen Punkte als Fehlergleichungen formulieren zu

$$-e_{\ell j} = -\Delta x_j - a_{12j}x_{\ell j} - a_{21j}y_{\ell j} + \bar{x}_{\ell}; p_{\ell j} \quad (13a)$$

$$-e_{\ell j} = -\Delta y_j - b_{12j}x_{\ell j} - b_{21j}y_{\ell j} + \bar{y}_{\ell}; p_{\ell j} \quad (13b)$$

oder

$$-e_{\ell j} = C_j'x_j' + C_j''x_j'' + \bar{x}_{\ell}; P_{\ell j} \quad (14)$$

Hier ist das Gesamtsystem (12), (14) zu lösen. Der Vektor x_j'' beinhaltet z_j ($z_j^T [a_{12},$

$a_{21}, b_{12}, b_{21}]_j$) und die radiometrischen Korrekturparameter. Die Punktkoordinaten $\bar{x}_{\ell}, \bar{y}_{\ell}$ treten als neue, zusätzliche Unbekannte auf. Die geometrischen Identitätsbedingungen werden hier auf eine Weise genutzt, die dem Vorgehen bei der Anblockmethode der Modellblockausgleichung entspricht. Die Gleichungen (13a), (13b) korrespondieren dort mit den Gleichungen für die Verknüpfungspunkte.

Wiewohl hier nur auf die Korrelation von Musterverbänden aus zwei Bildern Bezug genommen wurde, kann dieses Verfahren durchaus auf mehr Bilder angewendet werden. Durch Hereinnahme von Kollinearitätsgleichungen kann auch hier ein zusätzlicher Stabilisierungseffekt erzielt werden.

Ausblick

Das Anliegen dieses Artikels war es, verschiedene Möglichkeiten zur Ausnutzung geometrischer Zusatzbedingungen bei der kleinsten Quadrate Korrelation aufzuzeigen. Es wurden nur die grundlegenden Formeln wiedergegeben. Die Methoden sind noch im praktischen Einsatz zu überprüfen. Konvergenzverhalten, Rechenaspekte (Ausnutzung dünn besetzter Matrixstrukturen, Berechnungsmethoden für Designmatrizen und Normalgleichungen im Iterationsprozess) sowie Qualitätskriterien sind schwerpunktmässig zu untersuchen.

Die simultane Korrelation mit mehr als zwei Photos bietet sich nicht allein bei Off-line Anwendungen, sondern auch für On-line und Real-time Auswertungen an. Falls mehr als zwei Photos auf den Bühnen eines Analytischen Plotters Platz finden, ist es möglich, selbst bei

Verwendung nur zweier Arraykammern, mehr als zwei Korrelationsfenster sehr kurz hintereinander abzutasten. Ausserdem ist es durchaus vorstellbar, mehr als zwei Arraykammern simultan zum Abtasten einzusetzen. Auch bei der unmittelbaren Verwendung digitaler Kammern als Messkammern für Real-time Anwendungen werden sich Arrangements mit mehr als zwei Kammern in Zukunft durchsetzen.

Die Verfahren können auch in der terrestrischen Photogrammetrie verwendet werden.

Ein weiterer Schritt in Richtung verbesserter Image Matching Verfahren muss die Semantik des Bildes berücksichtigen. Methoden der Machine Vision und des Image Understanding werden somit künftig vermehrt Aufmerksamkeit finden.

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A Future for Realtime Photogrammetry

G. L. and T. B. Hobrough

1. Introduction

We are pleased and honoured to contribute to this volume commemorating the career of Professor Hellmut Schmid. There seems to be no limit to the growth and spread of ideas arising out of his pioneering work in the measurement of ballistic trajectories by photogrammetric means.

For several years the authors have been collaborating in the application of binocular stereo vision to industrial robots. The links connecting Professor Schmid's work and robot vision are the analytical stereo plotter and the auto-

mation of photogrammetric plotting by stereo image correlation.

The early work of G. L. Hobrough on single point stereo correlation for automating projection plotters was contemporary with U. V. Helava's formulation of the first analytical stereo plotting method (1956 to 1961). Helava's work in turn initiated the present flowering of analytical plotters, all of which derive in some measure from Schmid's fundamental ideas.

During the period 1967 to 1972 G. L. Hobrough developed the Gestalt Photomapper which was a high speed

plotter employing both area correlation and analytic methods. The Gestalt instrument produced high quality orthophotographs and digital topography but was limited to small map scales by its inability to accommodate height discontinuities.

Since 1972 the Hobroughs (G. L. & T. B.) have overcome the problem of height discontinuity, originally for the purpose of overcoming the scale limitation in automatic plotters but ultimately for enabling the application of stereo correlation to real-time stereo vision for the control of industrial robots.

The term "Stereopsis" is used in biology to denote binocular stereoscopic vision and is used here to denote the analogous process of binocular 3D vision for robots and automation. Our system is, in effect, a real-time short range photogrammetric system using TV images in place of photographs. Perhaps videogrammetry would be a better word when measurement is the chief object of the process.

2. The Need for Robot Stereopsis

Computer controlled robots for industrial automation have developed rapidly since 1970. Computer programming techniques have evolved rapidly allowing ever increasing complexity of function with ever decreasing setup time. Currently the provision of sensing means for the feedback control of motion is under intensive investigation. Most robots today are controlled by a digital computer which can store a large number of activity routines common to many industrial tasks. They are programmed to go through a fixed sequence of computations required for a particular activity. For example, an articulated arm robot may have in its memory a subroutine to produce straight line motion of a tool attached to an arm capable only of rotational movements at the joints. The ease of reprogramming robots for a nearly unlimited number of activities is the major reason for their success in industrial automation.

The addition of sensory means improves the performance of computer controlled robots in many ways. For example: Position and angle encoders provide a speed and accuracy of robot arm motion not possible by open loop methods. Force and touch sensors are used to control robot hands or "grippers" once contact with a work piece has been made.

Occasionally some physical attribute of a process can be used for sensing. For example the intense light emitted during arc welding can be tracked by a sensor and deviations of the weld position from its programmed position can be corrected in real time.

A general purpose remote sensing vision-like system is needed both for recognizing objects and for controlling their motion in space for robotic manufacturing operations.

The importance of artificial vision to the further development of robots is fully appreciated by the industry. Projections of the fraction of robots that will have vision by the end of the decade run as high as forty percent. Artificial vision research is active in many laboratories and several systems are available commercially. Most current systems are based on television techniques

using one camera and are therefore essentially two dimensional (2D).

2D artificial vision has been developed more for pattern recognition than for feedback control. It has been particularly successful with opaque flat objects viewed by back light from a supporting light table. The outline of their silhouettes is then traced by a suitable algorithm. Only the edge data is retained and from this the object can be classified and its position and orientation determined in two dimensions. The data processing is further simplified by the use of Boolean arithmetic in which only two states are possible, namely light and dark.

Elaborate non-Boolean software algorithms have also been developed, a typical application being to sort out objects of known shape from a cluster of objects using only the shading patterns provided by a single camera. Such algorithms allow a robot to identify and move objects under controlled conditions.

Finding shape information from shading data is a slow process and is usually impossible without prior knowledge of the object. Nevertheless, much study has gone into such matters and there is now a considerable body of knowledge for dealing with objects once the edges are defined. The resulting algorithms are now among the basic building blocks for artificial intelligence systems. Clearly, the control of operations in real-time and in real-space sorely needs a 3D vision system with video speed and resolution. A system such as Man's own binocular stereo apparatus that performs an analogous function every waking minute of our lives.

A Stereopsis System designed for assembly purposes needs the ability to locate, track and not lose sight of moving objects more than it needs recognition.

3. Outline of the Stereopsis System

The Stereopsis System currently under development by the authors consists of three separate physical modules: the camera head, the 3DC (three dimensional converter) and the application package.

The camera head contains two solid state video cameras scanned in exact synchronism plus the timing clock for the entire system. The optical/mechanical design of the camera head is adaptable to a variety of conditions such as work space size and distance (are we assembling watches, cars, or ships?), environmental concerns (is this workspace a clean room or a coal mine?) and so forth.

The lighting requirements for the Stereopsis System are essentially the same as for human workers. A moder-

ate level of soft general lighting with a somewhat higher level directed at the work. Such lighting reduces the darkness of shadows and enhances modeling and texture of the surfaces under observation.

The 3DC is a "black box" which takes in a pair of 2D video signals and converts them into a 3D digital model of the workspace. This is the electronic image correlation long familiar to photogrammetrists but unknown to the robotics industry until very recently. Additional outputs from the 3DC will include transformed video signals from the left and right cameras and a "change" signal to indicate wherever something has moved.

To provide the greatest possible utility to users equipment, the 3DC will deliver XYZ coordinate and change outputs in a form that promotes simple and reliable design for the post processing modules required for a variety of industrial automation tasks and artificial intelligence (AI) studies. For example, the transformed video signals will be delivered to a colour TV monitor to indicate the degree of image matching during initial set up and during experimental developments or AI studies. The application package will process the outputs from the 3DC to provide the robot with specific data about the workspace and the objects in it. Such processing falls into two distinct categories - global functions and local functions.

Global functions are those processing chores that are active at all times over the entire area of the scene. The primary global function is the sensing of edges or Z discontinuities. A derivative global function is the sensing gap closure which is used to control the contacting of objects and to avoid destructive collision.

Local functions are related to particular objects that have been "acquired" by an "object monitor". Examples of local functions are the acquiring of an object, the tracking of its position and velocity, and the computation of the object's centre and orientation.

An object monitor is a processor circuit that will acquire, store and track the outline of any selected object in the workspace. One object monitor card will be required for each object to be tracked but any number of object monitors can be in operation at the same time.

A more complex and specialized type of output processor will be used for inspection operations. In the case of large objects it will be necessary to look at the object section by section, to remember the shape and size of each section, to assemble all the data from the individual sections into one

coherent memory module and then to make a mathematical comparison of the inspected object with a standard of comparison stored in a separate part of memory. This process and its problems will sound familiar to the mapping photogrammetrist.

4. Performance Targets

The Stereopsis System will, like human stereopsis, provide nearly instantaneous identification of edges and shapes of objects. The location of edges is a central feature of our design and is used in many of the application modules. When combined with the vast repertory of algorithms designed for using the edge data laboriously derived from 2D imagery, stereo edge and shape detection will provide a tremendously powerful tool for artificial intelligence purposes.

A Stereopsis System will automatically tag the location of changes or movements in the field of view. This facility would be used, for example, to announce the arrival of a part which can then be tracked about the work station. Gap closure control provides a means for a robot to bring two parts close to each other with the intention of assembling them together. It controls the rate of closure for optimum impact on contact.

Related to gap closure is the "collision avoidance system". In effect, the collision avoidance system will keep track of all gaps between moving objects in the field of view and deliver to the robot a "time to first collision" signal. Thus, the robot will know if it is moving in a dangerous manner and how long it has to take corrective action.

We use the terms "immediately" and "online" in describing our Stereopsis System. A fair question would be, "How fast is immediate?"

The first Stereopsis System is being designed to provide a Z (or range) value for every pixel in the field of view every 17 milliseconds. Thus "immediate" means about 100 milliseconds when the system is first turned on and every 17 milliseconds thereafter. This is somewhat quicker than the equivalent human functions. The data rate could be increased by up to ten times at higher hardware cost or at lower resolution.

5. How Robot Stereopsis differs from Mapping Photogrammetry

Topographic photogrammetry is complicated by the freedom of the aerial camera to assume an arbitrary orientation in space at the moment of exposure. The recovery of camera orientation is a lengthy process requiring measurement of image differences (disparities) in two dimensions. Stereopsis

for robots can be much simpler in this respect since the relative position of the cameras is known. In many cases the cameras may be fixed together mechanically, eliminating entirely the necessity for relative orientation.

Stereo correlation for robots is complicated by the presence of large discontinuities in range at object edges and by rapid relative motion in the work space. On the other hand, it is simplified in comparison with photogrammetric correlation by the freedom to change viewpoints or camera spacing adaptively in response to work space requirements.

Electronic stereo correlation is greatly simplified when the two images are scanned in exact synchronism and in a direction precisely parallel to the base line. The camera axes should intersect each image surface at its centre and on corresponding scanning lines. Under these conditions X-parallax (dx) is uniquely determined by the time difference (dt) between the scanning of homologous image points in the left and right cameras:

$$dx = v(dt)$$

where v = the image scanning velocity and it can be shown that the range (Z) is determined by the relationship:

$$Z = \frac{fB}{dx} = \frac{fB}{v(dt)}$$

where f = principal distance (which may be significantly longer than focal length at close range)

B = camera spacing

Several optimizing adjustments of the fixed head system are possible. An arrangement for varying the camera spacing would enable the input geometry to be matched to size of work-space (care is necessary to maintain coplanarity of the image surfaces). Automatic control of aperture, focus, and convergence could be accomplished by adaptive loops using data available from the stereopsis system.

The simplified geometry of the prototype stereopsis system seems a far cry from the elegant complexity of analytic photogrammetry of Professor Schmid. This situation is unlikely to persist as the applications for stereopsis increase in size and complexity.

6. How Robot Stereopsis may further the Development of Mapping Systems

The automation of photogrammetric plotting has developed at a slow and sometimes unsteady pace for many years. The correlation of stereo images is the central function in such automation and has presented a series of technical problems requiring unique solutions with virtually no application

except in topographic mapping. It has been difficult to justify the investment required to develop solutions in view of the small number of instruments likely to be sold. Furthermore, there has been little activity in other industries potentially contributing to the cause of stereo correlation except for the tremendous advance in the computer technology.

The situation is now changing rapidly as robotization sweeps through the manufacturing industries of the world. The needs of robotic vision create a huge demand for new systems and methods, some of which may be useable in photogrammetric applications. For the first time, a wide variety of sophisticated yet inexpensive tools are becoming available to the designers of automatic stereo plotters and interpreters. We believe that the swelling wave of robotization will affect the mapping industry as profoundly as it is now affecting the automobile industry.

Technology now exists that can extend the performance of stereo image correlators for photogrammetric plotting to the physical limits of the input material and at a very high productivity. VLSI (Very Large Scale Integration) chips and the newer semi-custom devices greatly reduce the design time and equipment costs compared with earlier methods. The result will be a cost effectiveness in the next generation of automated photogrammetric instrumentation enormously higher than we have come to expect from the current generation.

7. Some Possibilities and Predictions

In this closing section we will make some predictions on the applications of real time photogrammetry and related subjects. Many (we hope not all) readers will find our suggestions far-fetched. Let us assure you that we present the following as serious possibilities and predictions.

7.1 Mapping

We believe that a very high speed aerial photogrammetry data reduction machine could be built that would produce an orthophoto, a digital model, an edge map and even an old fashioned contour sheet at scales of 1:1000 or 1:500 in 5 minutes or less per model. In fact, it may even be practical to do such a data reduction in less than 30 seconds.

Such a machine would practically eliminate the need for skilled plotter operators. It would create instead a need for skilled data editors, who would use an editing machine to view a contour map superimposed on the orthophoto to edit the contours to allow for vegetation and buildings. The editors could also view the edge map

superimposed on the orthophoto to edit or annotate all points on the terrain where the surface drops away at more than a selected angle (such as 89°?). Thus, people in photogrammetric process will be used only for tasks requiring judgement and knowledge and not taken up endlessly tracing out elevations and profiles.

7.2 Reconnaissance

We expect to see a variation of the editing machine suggested above for use in reconnaissance work. In such a reconnaissance machine a computer would compare highly detailed digital elevation models "before and after" to locate changes. All areas of change could be outlined on the orthophoto display for consideration by the operator. It should be practical to use uncontrolled models for both before and after and to locate all changes in 2-3 minutes per model.

7.3 Assembly Robots

The robot market seems likely to become about a hundred times the size

of the photogrammetric instrument market by 1990. The market for vision systems should be about US\$ 500,000,000 per year by then. We expect to see small and large robots with sophisticated stereopsis doing a multitude of industrial assembly tasks at speeds equal to or vastly exceeding human capabilities. Stereopsis would also assist and simplify the programming process in many ways.

7.4 Inspection Robotics

The detailed inspection of industrial products during the production process will be very much akin to the mapping photogrammetry of today. However the "terrain" being mapped may be only the size of a paper clip, the "camera stations" may be only a foot or two above the terrain and the system will have perhaps one tenth of a second to map the object, compare it with the standard and pronounce on its acceptability.

7.5 Navigation Robotics

At first automated factory vehicles will be able to drive about the warehouse

and factory using the same visual cues as a human driver. As the technology progresses it will not be long before such vehicles are used as automatic transfer and handling devices in remote or hazardous environments. We anticipate a tremendous political furor when they are first allowed onto the public streets and highways.

7.6 Medical

In the medical field we may come the full circle and a role reversal. Industry has been using the sight of man to guide machines for many years. Could not the design of machines perhaps guide man in his understanding of human visual processes?

If blind machines can be given sight, why not use machines to give sight to blind people?

Indeed, why not?

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Die Entwicklung der abtastenden, digitalen Bildaufnahme für die Photogrammetrie

O. Hofmann

Die photogrammetrische Messtechnik beruht auf dem Prinzip der räumlich zentralperspektiven Aufnahme und Auswertung photographischer Momentaufnahmen. Das Objekt wird von einem, oder bei stereoskopischer Arbeitsweise von zwei oder mehr Standpunkten aus in einer Momentaufnahme in die Bildebene eines Objektives, das den Zentralpunkt realisiert, abgebildet und photographisch-analog registriert. Mit den bekannten Mitteln der Ein- oder Zweibild-Photogrammetrie kann die geometrische Gestalt des aufgenommenen Objektes in beliebigem Massstab rekonstruiert werden. In der Zweibild-Photogrammetrie geschieht dies durch die Wiederherstellung der äusseren Orientierung der Messaufnahmen, d.h. der Bestimmung der räumlichen Positionen und Neigungen der Aufnahmekamera im Zeitpunkt der Momentaufnahme und der Bestimmung von Objektpunkten durch räumlichen Vorwärtsschnitt homologer Strahlen. Die Photogrammetrie benützt also zur Messung räumliche, zentralperspektive Strahlenbündel. Bemerkenswert dabei ist, dass die Wiederherstellung des Modells ohne Zuhilfenahme zusätzlicher, externer Messungen der Orientie-

rungsparameter oder von Objektpunkten in einem beliebigen Massstab bei beliebiger Modell-Orientierung gelingt. Sie geschieht allein mit Hilfe identischer Bildpunkte eines Objektes in den verschiedenen Bildern, deren homologe Strahlen zum Schnitt gebracht werden. In einem Aufsatz «Über den Wandel der geometrisch-algebraischen Modellvorstellung in der Photogrammetrie unter dem Einfluss computergestützter Auswerteverfahren» [13] hat H. Schmid den historischen Entwicklungsprozess dieser Orientierungsmethoden und der damit verbundenen Modellvorstellungen in prägnanter Form aufgezeigt. Das Modell mit seinem gesamten, in den Bildern enthaltenen Informationsinhalt ist invariant und kann mit Hilfe zusätzlicher Messungen, mindestens sieben, in ein übergeordnetes Koordinatensystem transformiert werden. H. Schmid formuliert die Aufgabe so: «Ganz allgemein ausgedrückt geht es

bei der Auswertung von photogrammetrischen Aufnahmen darum, eine bestimmte Anzahl von Punkten des zu vermessenden Objekts in einem vorgegebenen Koordinatensystem gemeinsam mit den Orten der Aufnahme und deren Orientierung zu bestimmen.» Schmid zeigt nun sehr anschaulich und klar formuliert, dass für die Problemlösung der historische Entwicklungsprozess und die zur Verfügung stehenden technischen Mittel massgebend waren. Der Umweg über die zuerst vorzunehmende Bestimmung der sog. relativen Orientierung – mathematisch durch die Koplanaritätsbedingungen ausdrückbar – und die anschliessende absolute Orientierung, d.h. die Transformation in das vorgegebene geodätische Koordinatensystem, ist durch den Einsatz optisch-mechanischer Stereo-Messgeräte bedingt. Dagegen wird die Benützung der Kollinearitätsgleichungen

$$\begin{aligned} x_{i,N} &= c \frac{a_{11}(X_i - X_N) + a_{21}(Y_i - Y_N) + a_{31}(Z_i - Z_N)}{a_{13}(X_i - X_N) + a_{23}(Y_i - Y_N) + a_{33}(Z_i - Z_N)} = F_x(p_N, k_i) \\ y_{i,N} &= c \frac{a_{12}(X_i - X_N) + a_{22}(Y_i - Y_N) + a_{32}(Z_i - Z_N)}{a_{13}(X_i - X_N) + a_{23}(Y_i - Y_N) + a_{33}(Z_i - Z_N)} = F_y(p_N, k_i) \end{aligned} \quad (1)$$