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Autor: Müller, Edgar

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On the Genesis of Movement and Time

EDGAR MÜLLER
Chemin des Bouleaux 14, 1012 Lausanne[i]

Summary

PAUL DIRAC's relativistic energy-momentum relationship $E^2 = m_0^2c^4 + \vec{p}^2c^2$ is a sum of four-squares. According to LEONHARD EULER, such a sum can be rewritten as a bilinear product of two sums of four squares each, $E^2 = m_0^2 + p_1^2 + p_2^2 + p_3^2 = (r_0^2 + r_1^2 + r_2^2 + r_3^2) \cdot (mo_0^2 + po_1^2 + po_2^2 + po_3^2)$. Sums of squares represent inner products of vectors with themselves. The equation $\vec{S} = \vec{R} * \overrightarrow{So}$, wherein the multiplication (*) is defined according to LEONHARD EULER's identity, and wherein $\vec{R} = (r_0, \ r_1, \ r_2, \ r_3)$ is a unitary operator with $r_0^2 + r_1^2 + r_2^2 + r_3^2 = 1$, describes the evolution of a previous physical system $\overrightarrow{So} = (mo_0, \ po_1, \ po_2, \ po_3)$ into a new physical system $\vec{S} = (m_0, \ p_1, \ p_2, \ p_3)$ without changing its energy E. LEONHARD EULER's identity turns out to be the *sine-qua-non-condition* for the existence of movement and time.

Résumé

La relation énergie – quantité de mouvement relativiste de PAUL DIRAC, $E^2 = m_0^2c^4 + \vec{p}^2c^2$ est une somme de quatre carrés. Selon LEONHARD EULER une telle somme peut être écrite comme le produit bilinéaire de deux sommes de quatre carrés: $E^2 = m_0^2 + p_1^2 + p_2^2 + p_3^2 = (r_0^2 + r_1^2 + r_2^2 + r_3^2) \cdot (mo_0^2 + po_1^2 + po_2^2 + po_3^2)$. Une somme de carrés représente un produit intérieur de vecteurs avec eux-mêmes. L'équation $\vec{S} = \vec{R} * \overrightarrow{So}$, où la multiplication (*) est définie selon l'identité de LEONHARD EULER, et où $\vec{R} = (r_0, \ r_1, \ r_2, \ r_3)$ est un opérateur unitaire avec $r_0^2 + r_1^2 + r_2^2 + r_3^2 = 1$, décrit l'évolution d'un système physique antérieur $\overrightarrow{So} = (mo_0, \ po_1, \ po_2, \ po_3)$ en un nouveau système physique $\vec{S} = (m_0, \ p_1, \ p_2, \ p_3)$ sans changement de son énergie E. L'identité de LEONHARD EULER apparaît comme la *condition sine-qua-non* pour l'existence du mouvement et du temps.

Zusammenfassung

PAUL DIRAC's relativistische Energie-Impuls Beziehung $E^2 = m_0^2c^4 + \vec{p}^2c^2$ ist eine Summe von vier Quadraten. Eine derartige Summe kann gemäss LEONHARD EULER als ein bilineares Produkt zweier Summen von je vier Quadraten geschrieben werden, $E^2 = m_0^2 + p_1^2 + p_2^2 + p_3^2 = (r_0^2 + r_1^2 + r_2^2 + r_3^2) \cdot (mo_0^2 + po_1^2 + po_2^2 + po_3^2)$. Quadratsummen stellen innere Produkte von Vektoren mit sich selbst dar. Die Gleichung $\vec{S} = \vec{R} * \overrightarrow{So}$, worin die Multiplikation (*) gemäss der EULERschen Identität definiert ist, und worin $\vec{R} = (r_0, \ r_1, \ r_2, \ r_3)$ einen Einheitsoperator mit $r_0^2 + r_1^2 + r_2^2 + r_3^2 = 1$ darstellt, beschreibt die Entwicklung eines vorgängigen physikalischen

Systems $\overrightarrow{So} = (mo_0, \ po_1, \ po_2, \ po_3)$ in ein neues physikalisches System $\vec{S} = (m_0, \ p_1, \ p_2, \ p_3)$ unter Erhaltung seiner Energie E. Die EULERsche Identität erscheint hier als die *sine-qua-non-Bedingung* für die Existenz von Bewegung und Zeit.

Introduction

The concepts of **space** and **time** are the fundamental framework for our physical measurements and our description of physical reality. *Special relativity* casts both into a unique 4-dimensional **space-time continuum**. Neither space, nor time, nor the relativistic space-time-continuum give the basis, however, for an accounting of quantities, nor for corresponding conservation laws. Speed has an asymptotic upper limit at the speed of light c, and space and time become strongly nonlinear in the neighborhood of this limit c.

On the other hand, energy and momentum are conserved physical quantities, allowing for quantitative accounting without upper limits. Physical reality is much easier described in terms of energy and momentum than in terms of space, speed and time. As shown below, the *space-time* of *Special Relativity* (i.e. the *MINKOWSKI space*) can be mathematically derived from the energy conservation law.

The *sine-qua-non condition* for the existence of movement

PAUL DIRAC's relativistic energy-momentum relationship[ii]

$$\frac{E^2}{c^2} - \vec{p}^{\,2} = m_0^2 c^2$$

can be rewritten in the form

$$\boldsymbol{E^2 = m_0^2 c^4 + \vec{p}^2 c^2} \qquad \textbf{(I)}$$

where $\vec{p} = (p_1, \ p_2, \ p_3)$ is the momentum vector in 3-dimensional space.

For particles at rest ($\vec{p} = 0$), this relationship reduces to ALBERT EINSTEIN's mass-energy-equivalence $E = m_0 c^2$ [iii].

The energy **E** appears here as the hypotenuse of a rectangle with sides $\boldsymbol{m_0 c^2}$ and $\boldsymbol{pc}$ (*Fig. 1*):

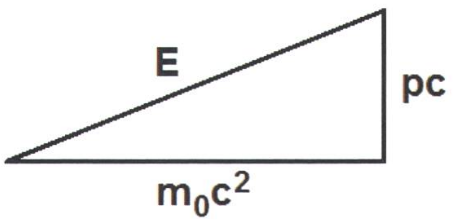


Figure 1: *Energy, momentum, and mass*

Energy E, rest mass m_0, and momentum p are thus expressions of one and the same physical reality, namely of **movement**. If we express energy E in [eV] (electron-volts), we may express momentum p in [eV/c], and rest mass m_0 or relativistic mass m in [eV/c^2].

Equation (I) can be further simplified if we measure distances in light-seconds instead of meters (i.e. by setting c = 1):

$$\boldsymbol{E^2 = m^2 = m_0^2 + \vec{p}^2 = m_0^2 + p_1^2 + p_2^2 + p_3^2} \qquad \textbf{(II)}$$

All quantities, i.e. **E**, $\mathbf{m_0}$ and **p** are now expressed in [eV].

The energy squared is a sum of four squares, $E^2 = m_0^2 + p_1^2 + p_2^2 + p_3^2$, which represents the inner (metric) product of a 4-dimensional momentum vector $\vec{S} = (m_0, \ p_1, \ p_2, \ p_3)$ with itself: $E^2 = (\vec{S})(\vec{S})$.

The rest mass m_0 appears as the fourth component of the relativistic momentum vector $\vec{\boldsymbol{S}}$, and the energy $\boldsymbol{E}$, which is equivalent to the relativistic mass $\boldsymbol{m}$[iv], is the total length (i.e. the absolute value) of said relativistic momentum vector, i.e. the total amount of movement in the physical system $(m_0, \ p_1, \ p_2, \ p_3)$ under consideration.

A *sine-qua-non condition* for the existence of movement (and thus of time) in the universe is that a given physical system $\vec{S} = (m_0, \ p_1, \ p_2, \ p_3)$ is the result of the evolution of a previous physical system, $\overrightarrow{So} = (mo_0, \ po_1, \ po_2, \ po_3)$ having the same energy E, i.e. the same amount of movement as $\vec{S}$. Otherwise movement would not be conserved and could not exist as a durable property in the universe; in other words, it would not exist at all[v].

The four-squares-identity provides the *sine-qua-non condition* of movement

According to LEONHARD EULER, **a sum of four squares can always be written as the bilinear product of two sums of four squares each** (*4-squares identity*)[vi]. Note that this holds as well for sums of two squares (*2-squares identity*)[vii] and for sums of eight squares (*8-squares identity*)[viii], but for no other sums of squares[ix] (see the attached **Appendix**).

Introducing a unitary vector operator $\vec{R} = (r_0, \ r_1, \ r_2, \ r_3)$,
with $r_0^2 + r_1^2 + r_2^2 + r_3^2 = 1$, we can rewrite equation (II) as:

$$\boldsymbol{E^2 = m_0^2 + p_1^2 + p_2^2 + p_3^2 = (r_0^2 + r_1^2 + r_2^2 + r_3^2) \cdot (mo_0^2 + po_1^2 + po_2^2 + po_3^2)} \ \textbf{(III)}$$

wherein:

$$\begin{aligned} m_0 &= (r_0 \cdot mo_0 - r_1 \cdot po_1 - r_2 \cdot po_2 - r_3 \cdot po_3) \\ p_1 &= (r_0 \cdot po_1 + r_1 \cdot mo_0 + r_2 \cdot po_3 - r_3 \cdot po_2) \\ p_2 &= (r_0 \cdot po_2 - r_1 \cdot po_3 + r_2 \cdot mo_0 + r_3 \cdot po_1) \\ p_3 &= (r_0 \cdot po_3 + r_1 \cdot po_2 - r_2 \cdot po_1 + r_3 \cdot mo_0) \end{aligned}$$

The proof of this is obtained by simple algebraic manipulation.
Note that these equations define a vector multiplication rule

$$\vec{\boldsymbol{S}} = \vec{\boldsymbol{R}} * \overrightarrow{\boldsymbol{So}} \qquad \textbf{(IV)}$$

through the inner product identities

$$E^2 = (\vec{S})(\vec{S}) = (\vec{R})(\vec{R}) \cdot (\overrightarrow{So})(\overrightarrow{So}) = (\vec{R} * \overrightarrow{So})(\vec{R} * \overrightarrow{So}) .$$

It is easily shown that $E^2 = m_0^2 + p_1^2 + p_2^2 + p_3^2 = mo_0^2 + po_1^2 + po_2^2 + po_3^2$, i.e., that the original physical system $\overrightarrow{So} = (mo_0, \; po_1, \; po_2, \; po_3)$ has the same energy E as the resulting physical system $\vec{S} = (m_0, \; p_1, \; p_2, \; p_3)$. The existence of internal degrees of freedom (represented by the operator $\vec{R}$), which preserve the total energy of the system (i.e., the length of the energy vector E representing the amount of movement) provides the mathematical conditions for the existence of movement and time[x] in the universe.

The operator $\vec{R}$ is a rotation operator of the form

$$\vec{R} = (cos(\varphi), \quad u_1 \sin(\varphi), \quad u_2 \sin(\varphi), \quad u_3 \sin(\varphi))$$

wherein $\vec{u} = (u_1, \; u_2, \; u_3)$ is a **unit vector** (a direction) in 3-dimensional space.
Its length squared evaluates to:

$$(\vec{R})(\vec{R}) = cos^2(\varphi) + (u_1^2 + u_2^2 + u_3^2) \cdot sin^2(\varphi) = 1.$$

The operator R thus provides a 2-dimensional manifold of unit vectors $\vec{u}$, representing the possible directions of movement in 3-dimensional space, and a 1-dimensional manifold of rotation angles φ, representing the possible amounts of movement between the fourth dimension (i.e. the energy or mass) and the said 3 space dimensions.

Seen from another point of view, the mathematical 4-squares-identity and the conservation of energy (movement) constitute the ultimate reasons for the 3-dimensionality of the physical space, which is spanned by the degrees of freedom of movement.

In the vector product (IV), $\vec{S} = \vec{R} * \overrightarrow{So}$, the first component

$$\boldsymbol{m}_{\boldsymbol{0}} = (\boldsymbol{r_0} \cdot \boldsymbol{mo_0} - \boldsymbol{r_1} \cdot \boldsymbol{po_1} - \boldsymbol{r_2} \cdot \boldsymbol{po_2} - \boldsymbol{r_3} \cdot \boldsymbol{po_3}) \qquad \textbf{(V)}$$

is an inner (metric) product in MINKOWSKI space, which shows the negative metric of *special relativity*, (+1,-1,-1,-1)[xi].

The MINKOWSKI space results here directly from EULER's four-squares-identity, i.e., it is obtained by mere algebraic means. One of the four dimensions (i.e. *degrees of freedom*) of MINKOWSKI space is used up by the condition of total energy conservation, leaving the remaining three dimensions for the *degrees of freedom* of movement.

Time

Considering a sequence of transformations leading from an original physical system $\overrightarrow{\boldsymbol{So}}$ to a final physical system $\overrightarrow{\boldsymbol{S_n}}$,

$$\overrightarrow{\boldsymbol{S_n}} = \overrightarrow{\boldsymbol{R_n}} * \overrightarrow{\boldsymbol{R_{n-1}}} * \dots * \overrightarrow{\boldsymbol{R_3}} * \overrightarrow{\boldsymbol{R_2}} * \overrightarrow{\boldsymbol{R_1}} * \overrightarrow{\boldsymbol{So}}\,,$$

the ordered chain of operators $\overrightarrow{\boldsymbol{R_\iota}}$ represents time, in the sense that this sequence of transformations can be compared to other sequences of transformations. A way of introducing time is to consider the rotation angle φ in the operator $\vec{R}$ to be a function of a steadily and homogeneously progressing variable **t**:

$$\vec{R} = (cos(\omega t),\quad u_1\sin(\omega t),\quad u_2\sin(\omega t),\quad u_3\sin(\omega t))\,.$$

Of course, the components of $\vec{u}$ can be functions of **t**, as well:

$$\overrightarrow{u(t)} = (u_1(t),\quad u_2(t),\quad u_3(t)), \text{ under the condition that } (u_1^2 + u_2^2 + u_3^2) = 1.$$

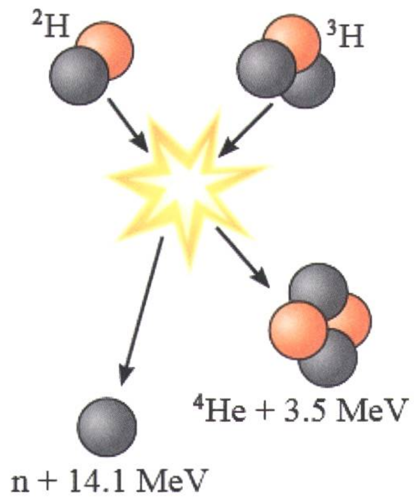
An illustration

The reaction of deuterium with tritium is exothermic, yielding an alpha particle and a neutron:

$$^2H + {}^3H \rightarrow {}^5He \rightarrow {}^4He + n + 17.6\ MeV$$

The deuterium and the tritium nuclei first combine to an unstable ^5He compound nucleus, which decays rapidly into an alpha particle and a neutron (*Fig. 2*).

The implied rest masses m_0 are as follows (NIST values):



Deuterium: $\mathbf{m_{0D}}$	**1'875.612 MeV/c²**
Tritium: $\mathbf{m_{0T}}$	**2'808.921 MeV/c²**
Total Educts (= $\mathbf{m_{0(5He)}}$)	4'684.533 MeV/c²
Helium: $\mathbf{m_{0He}}$	**3'727.379 MeV/c²**
Neutron: $\mathbf{m_{0n}}$	**939.565 MeV/c²**
Total Products:	4'666.944 MeV/c²
Mass defect:	17.589 MeV/c²

Figure 2: *The Deuterium-Tritium reaction*

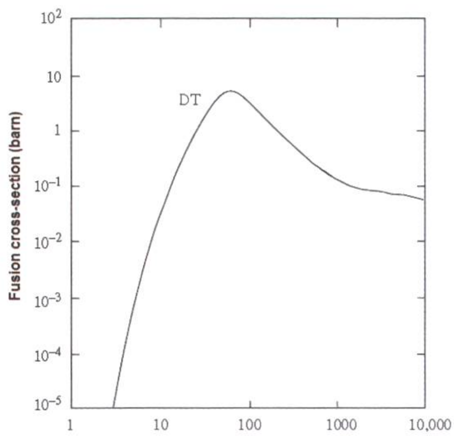


Figure 3: *Cross-section of the D-T*

The D-T-reaction has a maximum cross section of about 5 barn at a center-of-mass kinetic energy of 64 keV[xii] (*Fig. 3*); it is the easiest of all nuclear fusion reactions (if the term "*fusion*" is taken in the sense of "fusing together positively charged particles")

The reaction energy, corresponding to the mass defect, is carried away in about a 1 to 4 ratio by the helium nucleus and the neutron. This follows immediately from the classical energy- and momentum conservation laws[xiii]

$$2E_{kin} = \frac{p_{1He}^2}{m_{0He}} + \frac{p_{1n}^2}{m_{0n}} = p^2\left(\frac{1}{m_{0He}} + \frac{1}{m_{0n}}\right), \qquad \text{(VI)}$$

because the momenta p_{1He} and p_{1n} must be opposite and equal, and the rest mass m_{0He} of the helium nucleus is about four times the rest mass m_{0n} of the neutron. Of the liberated reaction energy (~17.6 MeV), 3.52 MeV are thus carried away by the helium nucleus, and 14.08 MeV by the neutron.

The ^5He decay is schematically depicted in *Fig. 4*; we only need to consider in this case the mass dimension (m_0) and one single momentum dimension (p_1):

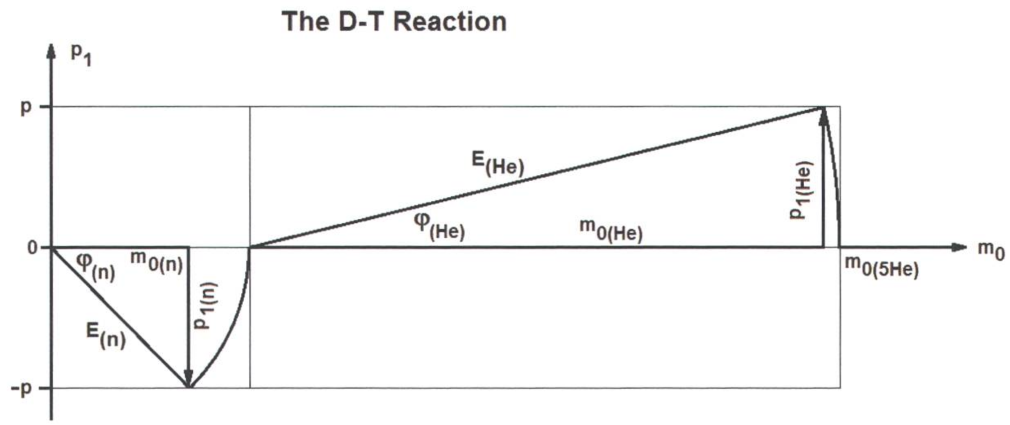


Figure 4: *Energy and Momenta of the D-T reaction*

Given that the compound nucleus ^5He is at rest, we have momentum compensation in the reaction products: $p_{1(He)} + p_{1(n)} = 0$.

The energy conservation, on the other hand, yields:

$$\sqrt{\left(m_{0(He)}\right)^2 + \left(p_{1(He)}\right)^2} + \sqrt{\left(m_{0(n)}\right)^2 + \left(p_{1(n)}\right)^2} = m_{0(5He)} \,.$$

The equation:

$$\sqrt{(3'727.379\ MeV)^2 + p^2} + \sqrt{(939.565\ MeV)^2 + p^2} = 4'684.533\ MeV$$

is solved by setting p = 162.965 MeV, which is the relativistic solution.

Hence, the mass of the compound nucleus [5]He is converted by two rotation operators $\overrightarrow{R_{(He)}} = \left(\cos\left(\varphi_{(He)}\right),\ \ sin\left(\varphi_{(He)}\right)\right)$ and $\overrightarrow{R_{(n)}} = \left(cos\left(\varphi_{(n)}\right),\ \ sin\left(\varphi_{(n)}\right)\right)$ into a helium nucleus $(m_{0(He)},\ \ p)$ and a neutron $(m_{0(n)},\ \ -p)$ having opposite equal momenta p.

The corresponding rotation angles are obtained as

$$\varphi_{(He)} = arctan\left(\frac{p}{m_{0(He)}}\right) = arctan\left(\frac{162.965\ MeV}{3727.379\ MeV}\right) = 2.50\ deg$$

$$\varphi_{(n)} = arctan\left(\frac{-p}{m_{0(n)}}\right) = arctan\left(\frac{-162.965\ MeV}{939.565\ MeV}\right) = -9.84\ deg$$

Conclusion

LEONHARD EULER's identity turns out to be the *sine-qua-non-condition* for the existence of movement and time in the universe. The four-square identity

$$\boldsymbol{E^2 = m_0^2 + p_1^2 + p_2^2 + p_3^2 = \left(r_0^2 + r_1^2 + r_2^2 + r_3^2\right) \cdot \left(mo_0^2 + po_1^2 + po_2^2 + po_3^2\right)}$$

provides an easy access to the description of physical transformations at the relativistic level.

Acknowledgement

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Appendix: The three n-squares identities

The 2-squares identity (DIOPHANTUS OF ALEXANDRIA):

$$(c_0^2 + c_1^2) = (a_0^2 + a_1^2)(b_0^2 + b_1^2)$$

where:

$$c_0 = a_0 b_0 - a_1 b_1$$
$$c_1 = a_0 b_1 + a_1 b_0$$

The 4-squares identity (LEONHARD EULER):

$$c_0^2 + c_1^2 + c_2^2 + c_3^2 = (a_0^2 + a_1^2 + a_2^2 + a_3^2)(b_0^2 + b_1^2 + b_2^2 + b_3^2)$$

where:

$$\begin{aligned}
c_0 &= (a_0 b_0 - a_1 b_1 - a_2 b_2 - a_3 b_3)\\
c_1 &= (a_0 b_1 + a_1 b_0 + a_2 b_3 - a_3 b_2)\\
c_2 &= (a_0 b_2 - a_1 b_3 + a_2 b_0 + a_3 b_1)\\
c_3 &= (a_0 b_3 + a_1 b_2 - a_2 b_1 + a_3 b_0)
\end{aligned}$$

The 8-squares identity (FERDINAND DEGEN):

$$(c_0^2 + c_1^2 + c_2^2 + c_3^2 + c_4^2 + c_5^2 + c_6^2 + c_7^2) =$$
$$(a_0^2 + a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 + a_6^2 + a_7^2)\,(b_0^2 + b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + b_6^2 + b_7^2)$$

where:

$$\begin{aligned}
c_0 &= (a_0 b_0 - a_1 b_1 - a_2 b_2 - a_3 b_3 - a_4 b_4 - a_5 b_5 - a_6 b_6 - a_7 b_7)\\
c_1 &= (a_0 b_1 + a_1 b_0 + a_2 b_3 - a_3 b_2 + a_4 b_5 - a_5 b_4 - a_6 b_7 + a_7 b_6)\\
c_2 &= (a_0 b_2 - a_1 b_3 + a_2 b_0 + a_3 b_1 + a_4 b_6 + a_5 b_7 - a_6 b_4 - a_7 b_5)\\
c_3 &= (a_0 b_3 + a_1 b_2 - a_2 b_1 + a_3 b_0 + a_4 b_7 - a_5 b_6 + a_6 b_5 - a_7 b_4)\\
c_4 &= (a_0 b_4 - a_1 b_5 - a_2 b_6 - a_3 b_7 + a_4 b_0 + a_5 b_1 + a_6 b_2 + a_7 b_3)\\
c_5 &= (a_0 b_5 + a_1 b_4 - a_2 b_7 + a_3 b_6 - a_4 b_1 + a_5 b_0 - a_6 b_3 + a_7 b_2)\\
c_6 &= (a_0 b_6 + a_1 b_7 + a_2 b_4 - a_3 b_5 - a_4 b_2 + a_5 b_3 + a_6 b_0 - a_7 b_1)\\
c_7 &= (a_0 b_7 - a_1 b_6 + a_2 b_5 + a_3 b_4 - a_4 b_3 - a_5 b_2 + a_6 b_1 + a_7 b_0)
\end{aligned}$$

References

[i] Author's address: EDGAR MÜLLER, Chemin des Bouleaux 14, 1012 Lausanne, Switzerland.

[ii] PAUL A. M. DIRAC: *The Quantum Theory of the Electron*. In: *Proceedings of the Royal Society of London. Series A*. Vol. 117, Nr. 778, January 1st, 1928, p. 610–624.

[iii] ALBERT EINSTEIN, *"Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?"*, Annalen der Physik, **18**: (1905), 639–643.

[iv] EUGENE HECHT *(2006). "There Is No Really Good Definition of Mass". Phys. Teach.* ***44*** *(1): 40–45.*

[v] This is PARMENIDES' argument for the durability of being, i.e. if something exists here and now, being must be stable in time. This argument is equally valid for the durability of movement: if movement exists here and now, the quantity of movement in the universe must be stable in time.

[vi] Letters CXV and CXXV of LEONHARD EULER to CHRISTIAN GOLDBACH.

[vii] DIOPHANTUS OF ALEXANDRIA, *Arithmetica* (III, 19).

[viii] FERDINAND DEGEN (about1818), later independently rediscovered by JOHN THOMAS GRAVES (1843) and by ARTHUR CAYLEY (1845).

[ix] ADOLF HURWITZ's theorem states that $(a_1^2 + a_2^2 + a_3^2 + \ldots + a_n^2)(b_1^2 + b_2^2 + b_3^2 + \ldots + b_n^2) = c_1^2 + c_2^2 + c_3^2 + \ldots + c_n^2$, wherein the c_i are bilinear functions of the a_i and the b_i, is only possible for n = 1 (trivial case), 2, 4, and 8. ADOLF HURWITZ *(1898), "Über die Composition der quadratischen Formen von beliebig vielen Variabeln", Goett. Nachr.: 309–316.*

[x] Time can be looked at as a measure of movement. A clock is a reference movement.

[xi] Note that also the 2-squares identity gives rise to a space with negative metrics (+1,-1), implying a single dimension of movement, and that the 8-squares identity gives rise to a space with negative metrics (+1,-1,-1,-1,-1,-1,-1,-1) implying 7 dimensions of movement.

[xii] STEFANO ATZENI, *"Nuclear* Fusion Reactions", chapter one in S. ATZENI and J. MEYER-TER-VEHN: "*The Physics of Inertial Fusion*", Oxford University Press (2004,2009).

[xiii] At the resulting particle energies, the relativistic effects are still small.