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The massive particle as a quantum-locked state of movement

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Summary

According to the relativistic energy invariant, the rest mass m_0 is a fourth, locked component of movement, orthogonal to the free components p_x, p_y, p_z. From this it must be concluded that m_0 is a quantum-locked state of movement, determined by the spectrum $\Box\Psi = \lambda\Psi$ of the D'ALEMBERT operator.

Résumé

D'après l'invariance d'énergie relativiste, la masse au repos m_0 est une quatrième composante du mouvement, verrouillée et orthogonale aux composantes p_x, p_y, p_z, qui à leur tour sont libres. On doit en conclure que m_0 est un état quantique verrouillé du mouvement, déterminé par le spectre $\Box\Psi = \lambda\Psi$ de l'opérateur de D'ALEMBERT.

Zusammenfassung

Aus der relativistischen Energie-Invarianz folgt, dass die Ruhemasse m_0 eine vierte Komponente der Bewegung ist, verriegelt und senkrecht zu den Komponenten p_x, p_y, p_z, die ihrerseits frei sind. Man muss daraus schliessen dass m_0 ein verriegelter Quantenzustand der Bewegung ist, bestimmt durch das Spektrum $\Box\Psi = \lambda\Psi$ des D'ALEMBERT'schen-Operators.

As shown in a previous paper[i], physical reality is conditioned by the algebraic 2-squares, 4-squares, and 8-squares identities. These identities are mathematical singularities, which give rise to the complex numbers, the quaternions, and the octonions, respectively. In quaternion space $\mathbb{H}$, which is the physical 4-space with negative metric signature (+1,-1,-1,-1), POISSON'S equation becomes a wave equation[ii]:

$$\Delta(\varphi(x)) = \frac{\partial^2\varphi}{\partial {x_0}^2} - \frac{\partial^2\varphi}{\partial {x_1}^2} - \frac{\partial^2\varphi}{\partial {x_2}^2} - \frac{\partial^2\varphi}{\partial {x_3}^2} = \rho(x). \quad (1)$$

In its most general form, this wave equation represents the *fundamental equation of electrodynamics*, which is the LORENTZ-invariant form of MAXWELL'S equations:

$$\Box A = \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial {x_1}^2} - \frac{\partial^2}{\partial {x_2}^2} - \frac{\partial^2}{\partial {x_3}^2}\right) A = \mu_0 J, \quad (2)$$

wherein $\Box$ represents D'ALEMBERT's operator ; $A = \left(\frac{\varphi}{c}, (A_1, A_2, A_3)\right)$ the 4-potential, composed of scalar and vector potential, and $J = (\rho c, (J_1, J_2, J_3))$ the 4-current density, composed of charge and current.

Equation (2) reflects the structure of the relativistic energy invariant:

$$\frac{E^2}{c^2} - \vec{p}^2 = {m_0}^2 c^2. \quad (3)$$

If we measure distance in light-seconds[iii] instead of meters, c equals 1, and the relativistic energy invariant simplifies to: $E^2 - \vec{p}^2 = {m_0}^2$, or $E^2 = {m_0}^2 + \vec{p}^2$.

From this equations it is immediately evident that moment $\vec{p} = (p_x, p_y, p_z)$ and rest mass m_0 are mutually orthogonal components of the total energy **E**, which means that **the rest mass m_0 is a fourth component of movement**.

Whereas we experimentally know that the components of movement p_x, p_y, p_z may take arbitrary values, we also experimentally know that the rest mass m_0 is tied to determined discrete values, i.e. the rest masses of the known elementary particles and of their possible compounds[iv].

From this it must be concluded that **the rest mass m_0 is determined by the spectrum[v] of the D'ALEMBERT operator $\Box$**:

$$\Box\Psi = \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial {x_1}^2} - \frac{\partial^2}{\partial {x_2}^2} - \frac{\partial^2}{\partial {x_3}^2}\right) \Psi = \lambda\Psi, \quad (4)$$

wherein the variable **Ψ is a quaternion wave-function expressing movement in 4-space**.

In particle physics, the octonion equivalent of this formula must ultimately also be considered:

$$\bigcirc\Psi = \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial {x_1}^2} - \frac{\partial^2}{\partial {x_2}^2} - \frac{\partial^2}{\partial {x_3}^2} - \frac{\partial^2}{\partial {x_4}^2} - \frac{\partial^2}{\partial {x_5}^2} - \frac{\partial^2}{\partial {x_6}^2} - \frac{\partial^2}{\partial {x_7}^2}\right) \Psi = \lambda\Psi, \quad (5)$$

wherein **ψ is an octonion wave function expressing movement in 8-space[vi]**.

The massive particle therefore appears as a quantum-locked state of movement in four or eight dimensions. It has previously been shown by DIRAC that this holds for the electron / positron in 4-space:

Setting the scalar $\boldsymbol{\lambda} = -\frac{m^2c^2}{\hbar^2}$ and taking a scalar wave function ψ instead of a quaternion function $\boldsymbol{\Psi}$, equation (4) goes over into the well-known KLEIN-GORDON[vii] equation:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi - \nabla^2\psi + \frac{m^2c^2}{\hbar^2}\psi = 0. \tag{6}$$

This equation was the starting point for DIRAC'S[viii] derivation of a quantum mechanical description of the electron, which factors the operator and obtains a first-order differential equation for the electron's wave function:

$$\left(\beta mc^2 + c(\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3)\right)\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}. \tag{7}$$

The factors α_1, α_2, α_3, and β are 4x4 matrices with the multiplicative properties required to factor equation (6).

Equations (4) and (5) can also be factorized[ix] into first-order differential equations using *DIRAC's coup*, which makes the spectra of eigenvalues and the corresponding eigenfunctions of the D'ALEMBERT operator in quaternion 4-space, and their homologues in octonion 8-space, accessible to calculation.

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REFERENCES

[i] EDGAR MÜLLER, *De la réalité des nombres*, Bull. Soc. Frib. Sc. Nat. Vol. **103** (2014) p. 83-90

[ii] Note that ρ(x) changes sign in a space with negative metrics, because of GAUSS' law: In positive metrics GAUSS' law states that $div(E) = \partial E_x/\partial x + \partial E_y/\partial y + \partial E_z/\partial z = -\rho$. In negative metrics (in quaternion or MINKOWSKI space) E becomes (iE_x, jE_y, kE_z), and *div* becomes $\left(\frac{i\partial}{\partial x}, \frac{j\partial}{\partial y}, \frac{k\partial}{\partial z}\right)$, with $i^2 = j^2 = k^2 = ijk = -1$, which changes the sign of ρ in the formula. The same holds for the fundamental equation of electrodynamics, where $\mu_0 J$ is positive.

[iii] After the invention of the *optical frequency comb*, which allows a direct measurement of the frequency of light waves, the *meter* has ceased to be a primary unit of physics; it is now merely defined via the product of time and light speed c: 1 nano-light-second (*nls*) being a convenient unit for daily use, equaling about 30 cm.

[iv] For a compound there is a mass lack corresponding to the binding energy of the components.

[v] LYNN A. STEEN, Highlights in the history of spectral theory, *The American Mathematical Monthly*, vol. 80, 1973, pp. 359-381.

[vi] In analogy to the *Quabla* operator □ in 4-space, we use the *Octogon* ○ as a symbol for the corresponding operator in 8-space.

[vii] OSKAR KLEIN, *Z. Phys.* , **37** (1926) pp. 895–906; WALTER GORDON, *Z. Phys.* , **40** (1926–1927) pp. 117–133.

[viii] PAUL A. M. DIRAC, The Quantum Theory of the Electron; *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, Vol. **117**, No. 778 (Feb. 1, 1928), pp. 610-624.

[ix] The amenability to factoring is obvious from the multiplicative norm holding in quaternion and in octonion space: a sum of four, respectively eight squares can always be written as a bilinear product of two sums of four, respectively eight squares, where both factors may be chosen equal; hence the square root of the operator may be taken.