Other examples should be tested to confirm, or invalidate, $d_3$ as a rightful analogue of prime natural density in $\mathbf{Z}$.

We conclude by a general question pertaining to the density theory in number systems presented in the book [Bu]. Consider a class $\mathcal{K}$ of finite structures and a property $\mathcal{P}$. Define, for each $n \geq 1$, $\mathcal{K}_n$ as the subset of these structures of size $n$. Define $p_n$ as the proportion of structures in $\mathcal{K}_n$ that have property $\mathcal{P}$, and $P_n$ as the proportion of structures in $\bigcup_{k=1}^{n} \mathcal{K}_k$ that have property $\mathcal{P}$. In [Bu], the function $p_n$ (resp. $P_n$) is referred to as the *local* (resp. *global*) *counting function* and is somewhat akin to the $d_1$ (resp. $d_2$) approximant of order $n$. General results [Bu] have been proved in which the existence of a limit for $p_n$, or $P_n$, as $n \to \infty$ has been established. Suppose we define an *average counting function* $\bar{p}_n$ as $\frac{1}{n} \sum_{k=1}^{n} p_k$. Are there classes of structures such that neither $p_n$, nor $P_n$ have a limit law, but $\bar{p}_n$ does ? Can we build a general theory of limit laws based on the function $\bar{p}_n$ ? This function being similar to the $d_3$-approximant of order $n$, Propositions 1.11 and 1.13 of our paper suggest that if $\bar{p}_n$ converges there are general hypotheses under which the associated Dirichlet density also exists.

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