

# Proper actions on acyclic spaces

Autor(en): **Leary, Ian J.**

Objektyp: **Article**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **54 (2008)**

Heft 1-2

PDF erstellt am: **24.09.2024**

Persistenter Link: <https://doi.org/10.5169/seals-109917>

## **Nutzungsbedingungen**

Die ETH-Bibliothek ist Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Inhalten der Zeitschriften. Die Rechte liegen in der Regel bei den Herausgebern.

Die auf der Plattform e-periodica veröffentlichten Dokumente stehen für nicht-kommerzielle Zwecke in Lehre und Forschung sowie für die private Nutzung frei zur Verfügung. Einzelne Dateien oder Ausdrucke aus diesem Angebot können zusammen mit diesen Nutzungsbedingungen und den korrekten Herkunftsbezeichnungen weitergegeben werden.

Das Veröffentlichen von Bildern in Print- und Online-Publikationen ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Die systematische Speicherung von Teilen des elektronischen Angebots auf anderen Servern bedarf ebenfalls des schriftlichen Einverständnisses der Rechteinhaber.

## **Haftungsausschluss**

Alle Angaben erfolgen ohne Gewähr für Vollständigkeit oder Richtigkeit. Es wird keine Haftung übernommen für Schäden durch die Verwendung von Informationen aus diesem Online-Angebot oder durch das Fehlen von Informationen. Dies gilt auch für Inhalte Dritter, die über dieses Angebot zugänglich sind.

## 48

### PROPER ACTIONS ON ACYCLIC SPACES

by Ian J. LEARY

Here are a few questions about proper cellular actions of discrete groups  $G$  on acyclic spaces. I have deliberately avoided the classifying space for proper  $G$ -actions,  $\underline{E}G$ , partly because some of the questions have already been answered for this space, and partly because I know that some other people will write in this volume about questions concerning  $\underline{E}G$ . I start with a version of the classic question that was posed by Ken S. Brown [1], p.226.

QUESTION 48.1. *If  $G$  is of finite virtual cohomological dimension, does  $G$  act properly on some acyclic space of dimension equal to  $\text{vcd } G$  ?*

REMARK 48.2. If  $\text{vcd } G$  is not equal to 2, then ‘acyclic’ in the above question can be replaced by ‘contractible’ without changing the question. The answer is ‘yes’ when  $\text{vcd } G = 1$  by a theorem of Martin Dunwoody [2], and Quillen’s plus construction can be used to replace an acyclic space of dimension  $n$  by a contractible space of dimension equal to the maximum of  $n$  and 3.

Brita Nucinkis and I found examples to show that the dimension of the space  $\underline{E}G$  can be strictly greater than  $\text{vcd } G$  [5]. Some of the techniques that we used in [5], including Bredon cohomology, were learned from Guido Mislin.

Secondly, a rather vague question. It is well known that  $\text{vcd } G$  is finite if and only if  $G$  is virtually torsion-free and  $G$  acts properly on some finite dimensional contractible space [1].

QUESTION 48.3. *Are there any results concerning group cohomology where virtual torsion-freeness plays a role? For example, are there any results about  $H^*(G; \mathbf{Z}G)$  that hold for groups of finite vcd, but do not hold for all groups in Peter Kropholler's class  $\mathfrak{H}_1\mathfrak{F}$ ?*

Peter Kropholler's class  $\mathfrak{H}_1\mathfrak{F}$  consists of the groups that admit a proper action on some finite-dimensional contractible CW-complex. (See [3] for further details and for the definition of the larger class  $\mathfrak{H}\mathfrak{F}$ .)

Finally, a few questions concerning the connection between algebraic and topological finiteness conditions. See also [4], [5].

QUESTION 48.4. *If  $G$  is of type FP over a ring  $R$ , does  $G$  act cellularly cocompactly on some  $R$ -acyclic CW-complex  $X$  with stabilizers whose orders are units in  $R$ ?*

There is an algebraic version of this question too. Define a *projective permutation module* for the group algebra  $RG$  to be a direct sum of modules isomorphic to  $RG/H$ , where  $H$  ranges over the finite subgroups whose orders are units in  $R$ . Say that  $G$  is of type FPP over  $R$  if there is a finite resolution of  $R$  over  $RG$  by finitely generated projective permutation modules.

QUESTION 48.5. *If  $G$  is FP over  $R$ , is  $G$  necessarily of type FPP over  $R$ ?*

For  $R = \mathbf{Z}$ , this question is equivalent to the famous question of whether every group of type FP is FL.

QUESTION 48.6. *If  $G$  is FL over a prime field  $F$ , does  $G$  act freely cellularly cocompactly on some  $F$ -acyclic CW-complex?*

REMARK 48.7. There are groups that are FP but not FL over  $\mathbf{Q}$ , and are FL over  $\mathbf{C}$  [4].

Such a group cannot act freely cellularly cocompactly on any  $\mathbf{C}$ -acyclic CW-complex. It is because of these examples that the previous question is stated only for the fields  $\mathbf{Q}$  and  $\mathbf{F}_p$ .

## REFERENCES

- [1] BROWN, K. S. *Cohomology of Groups*. Graduate Texts in Mathematics 87. Springer-Verlag, 1982.
- [2] DUNWOODY, M. J. Accessibility and groups of cohomological dimension one. *Proc. London Math. Soc. (3)* 38 (1979), 193–215.
- [3] KROPHOLLER, P. H. On groups of type  $FP_\infty$ . *J. Pure Appl. Algebra* 90 (1993), 55–67.
- [4] LEARY, I. J. The Euler class of a Poincaré duality group. *Proc. Edinb. Math. Soc.* 45 (2002), 421–448.
- [5] LEARY, I. J. and B. E. A. NUCINKIS. Some groups of type VF. *Invent. Math.* 151 (2003), 135–165.

Ian J. Leary

Department of Mathematics  
The Ohio State University  
231 West 18th Avenue  
Columbus, Ohio 43210  
USA  
*e-mail*: leary@math.ohio-state.edu