Isomorphism conjectures for the mapping class group

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ISOMORPHISM CONJECTURES FOR THE MAPPING CLASS GROUP

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I enjoyed Guido's lecture in Münster where he showed that Teichmüller space is a classifying space for actions of the mapping class group whose isotropy groups are all finite. I will pose a question and then recall one I posed at that time.

Let Σ_g be a closed surface of genus g, let Γ_g be its mapping class group, and $\mathcal{T}_g \cong \mathbf{R}^{6g-6}$ its Teichmüller space.

QUESTION 26.1 (A special case of the Borel Conjecture). *Let Γ be a torsion free subgroup of the mapping class group, for example the Torelli group. Is the action of Γ on $\mathcal{T}_g$ topologically rigid? That is, is any proper homotopy equivalence of manifolds $h \colon M \to \mathcal{T}_g/\Gamma$, which is a homeomorphism outside of a compact set, homotopic to a homeomorphism, via a homotopy which is constant outside of a compact set?*

QUESTION 26.2 (Isomorphism conjecture injectivity). *Is there a contractible compactification of Teichmüller space which is small at infinity and equivariant with respect to the action of the mapping class group?*

To say a compactification $\overline{\mathcal{T}}_g$ of $\mathcal{T}_g$ is *small at infinity* means that for every compact subset K of Teichmüller space, for every boundary point $y \in \overline{\mathcal{T}}_g - \mathcal{T}_g$, and for every neighborhood U of y in the compactification $\overline{\mathcal{T}}_g$, there is a smaller neighborhood V so that for every $\gamma \in \Gamma_g$, if $\gamma K \cap V$ is nonempty, then $\gamma K \subset U$.

DISCUSSION. A solution to Question 26.1 would likely involve carrying out the program of Farrell–Jones [1] in the mapping class group case. A positive solution to Question 26.2 would likely lead to a proof of the injectivity map of the assembly map in K- and L-theory with respect to the family of finite subgroups (by modifying Rosenthal's thesis [3]), and thereby a new proof of the Novikov conjecture in this case. (It seems that the Novikov conjecture in the mapping class group case has been recently proved by Ursula Hamenstädt [2].)

REFERENCES

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