by I of elements of $\mathcal{F}_1$ is a family of concentric circles, since it has $T(W_1)$ as associated subspace. It follows that $I(\mathcal{F}_2)$ is a family of straight lines through the common center of the circles of $\mathcal{F}_1$. $\square$

5.3 A FINAL REMARK ON THE CASE $n = 2$.

To conclude, we observe that if a conformal map $f\colon U \to \mathbf{R}^2$ as in Theorem 2 has the property that *every* segment of straight line contained in U is mapped by f to a piece of circle or straight line, then it is given as in the statement of Liouville's theorem. For, by the discussion in the previous section, under this assumption Lemma 6 holds for $F = \mathcal{I}_{p_0,w,A}(f)\colon U \to \mathbf{V}^3 \subset \mathbf{L}^4$, and hence the remaining part of the proof of Theorem 5 also applies.

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