In case $k = \mathbf{R}$, Theorem 4.3 is due to Zorn [29].

The proofs of Proposition 2.3 and Theorem 4.1 in the special case $k = \mathbf{R}$ are strongly based on the topology of $\mathbf{R}^n$, and can therefore not easily be initiated for arbitrary real closed fields. Instead, Tarski's method enables us to use these results without considering their proofs.

The question arises to what extent Tarski's method can be used to deduce results on other classes of algebras over real closed fields, and possibly even over other types of ground fields. To our knowledge, no systematic attempt in this direction has hitherto been made.

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