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ON AN ASSERTION IN RIEMANN'S HABILITATIONSVORTRAG

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ABSTRACT. We study an assertion in Riemann's Habilitation Lecture of 1854. Namely, the determination of the metric given $n\frac{n-1}{2}$ sectional curvatures.

1. INTRODUCTION

Modern differential geometry was born with Riemann's Habilitation Lecture "Ueber die Hypothesen, welche der Geometrie zu Grunde liegen" (*On the Hypotheses which lie at the Foundations of Geometry*) of 1854 at Göttingen [R], [We]. In this lecture Riemann defines the curvature tensor R. One says that M is *flat* if M is locally isometric to $\mathbf{R}^n$ with the usual metric; the tensor R vanishes if and only if the metric is flat. M. Spivak [Sp1] translates Riemann's Lecture and explains it in modern terms. Let

$$Q(X,Y) := \frac{\langle R(X,Y)\,Y, X\rangle}{|X \wedge Y|^2}$$

be the sectional curvature. Spivak [Sp1, p. 4B-25], [Sp2, p. 176] makes the following

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