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IDEAL SOLUTIONS OF THE TARRY-ESCOTT PROBLEM OF DEGREE FOUR AND A RELATED DIOPHANTINE SYSTEM

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ABSTRACT. In this paper, the complete ideal symmetric solution in integers of the Tarry-Escott problem of degree four, that is, of the system of simultaneous equations $\sum_{i=1}^{5} a_i^r = \sum_{i=1}^{5} b_i^r$, $r = 1,2,3,4$, has been obtained. In addition, a parametric ideal non-symmetric solution has also been obtained, and this non-symmetric solution has been used to obtain a parametric solution of the diophantine system $\sum_{i=1}^{5} a_i^r = \sum_{i=1}^{5} b_i^r$, $r = 1,2,3,4$ and 6.

1. INTRODUCTION

The Tarry-Escott problem of degree k consists of finding two sets of integers $a_1, a_2, \ldots, a_s$ and $b_1, b_2, \ldots, b_s$ such that

$$\sum_{i=1}^{s} a_i^r = \sum_{i=1}^{s} b_i^r, \qquad r = 1,2,\ldots,k. \tag{1}$$

There is a well-known theorem [6, p. 614] due to Frolov according to which the relations (1) imply that

$$\sum_{i=1}^{s} (Ma_i + K)^r = \sum_{i=1}^{s} (Mb_i + K)^r, \qquad r = 1,2,\ldots,k, \tag{2}$$

where M and K are arbitrary integers. That is, if $(a_1, a_2, \ldots, a_s; b_1, b_2, \ldots, b_s)$ is a solution of the system (1), then

$$(Ma_1 + K, \ldots, Ma_s + K; Mb_1 + K, \ldots, Mb_s + K)$$

is also a solution of (1). This theorem is easily established by using the binomial theorem. If one solution of the system (1) is obtained from another through