THEOREM 4.16. *Let $n \in \mathbf{Z}$, $n \geq 1$. Then for all χ and for all $t \in \mathbf{C}_p$, $|t|_p < 1$,*

$$B_{-n,\chi}(-t) = (-1)^n \chi(-1) B_{-n,\chi}(t) .$$

Proof. Since

$$B_{-n,\chi}(t) = \sum_{m=0}^{\infty} \binom{-n}{m} B_{-n-m,\chi} t^m ,$$

and $B_{-n-m,\chi} = 0$ whenever $n+m \not\equiv \delta_\chi \pmod 2$ for each $m \in \mathbf{Z}$, $m \geq 1$, we see that $B_{-n,\chi}(t)$ is either an odd or an even function according to whether $n + \delta_\chi$ is odd or even, respectively. Thus

$$\begin{aligned} B_{-n,\chi}(-t) &= (-1)^{n+\delta_\chi} B_{-n,\chi}(t) \\ &= (-1)^n \chi(-1) B_{-n,\chi}(t) , \end{aligned}$$

and the proof is complete. □

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Glenn J. Fox

Department of Mathematics
and Computer Science
Emory University
Atlanta, GA 30322
U. S. A.
e-mail: fox@mathcs.emory.edu