Secondly, Birkhoff's Theorem also holds true for invariant curves of products $\phi_N \circ \cdots \circ \phi_1$ of monotone twist mappings *of the same sign*. In general, such products are not monotone twist mappings anymore. This generalization follows immediately by our method if, even more generally, each ϕ_n satisfies the same "cone condition" on $(\phi_{n-1} \circ \cdots \circ \phi_1)(\Gamma)$. For every single ϕ_n presses more area into a fold, and $\sup_{n \geq 0} |\Omega_n| < \infty$ because Γ is mapped onto itself again after N steps, instead of one. A proof along the traditional lines was given by Mather only a couple of years ago [Ma3, Appendix].

Finally, we did not really need that ϕ is a diffeomorphism. Everything can also be formulated and proved for homeomorphisms that preserve Lebesgue measure and satisfy the "cone condition".

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