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a normal subgroup of  $H$  then  $D(\gamma)$  is in this subgroup (note that there are finitely many such normal subgroups).

Since  $D(\gamma)$  has infinite order (if  $\gamma$  is non-trivial),  $\langle D(\gamma) \rangle$  has positive dimension so that it contains a non-trivial one-parameter group. Hence every non trivial semi-simple element in  $D(\Gamma)$  yields a one-parameter group contained in  $H_0$ . We now show that these one-parameter subgroups generate the connected component of the identity in  $H$ . Observe the following elementary fact: if a family of vectors spans the Lie algebra of a Lie group, then the one-parameter groups generated by these vectors generate the connected component of the identity. Therefore, we consider the linear span  $\mathfrak{E}$  in the Lie algebra  $\mathfrak{H}$  of  $H$  of the Lie algebras of all the subgroups  $\langle D(\gamma) \rangle$  for  $\gamma$  semi-simple. It is enough to show that  $\mathfrak{E} = \mathfrak{H}$ . Note that  $\mathfrak{E}$  is certainly non-trivial since semi-simple elements are Zariski dense in  $H$ . Note also that  $\mathfrak{E}$  is invariant under the adjoint action of  $D(\Gamma)$ , hence under the adjoint action of  $H$  since  $D(\Gamma)$  is Zariski dense in  $H$ . It follows that  $\mathfrak{E}$  coincides with the product of some of the simple factors of  $\mathfrak{H}$ . The only possibility is that  $\mathfrak{E} = \mathfrak{H}$  since otherwise, all the semi-simple  $D(\gamma)$  would have some power contained in the same product of some but not all of the simple factors of  $H$  (note that the algebraic Abelian group  $\langle D(\gamma) \rangle$  has a finite number of connected components). This implies that all semi-simple elements of  $D(\Gamma)$  are contained in some non trivial normal subgroup of  $H$ . This is not possible by the following argument. In the algebraic group  $H$ , there is a non-empty open Zariski set consisting of semi-simple elements which are not contained in any non-trivial normal subgroup of  $H$ . Since  $D(\Gamma)$  is Zariski dense in  $H$ , it intersects non-trivially this open set.

It follows that  $H_0$  contains the connected component of the identity of  $H$ . Therefore  $H_0$  is a semi-simple Lie group of finite index in  $H$ . By Kushnirenko's theorem, we can analytically linearize  $\phi(H)$  (one also uses Remark 2.1) and in particular  $\Phi(\Gamma)$ .

Theorem 10.4 is proved.

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