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COMMISSION INTERNATIONALE DE L'ENSEIGNEMENT MATHÉMATIQUE

(THE INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION)

ICMI STUDY ON THE TEACHING AND LEARNING OF MATHEMATICS AT UNIVERSITY LEVEL

DISCUSSION DOCUMENT

The purpose of this Discussion Document is to raise important issues related to the study of the teaching and learning of mathematics at university level and to stimulate discussion and research on these topics as background for a conference to be held in Singapore in December 1998. After this conference, a publication covering the fundamental areas of the topic will be published in the *ICMI Study Series*. The main aspects of the Study will also be presented at ICME–9 in Makuhari, Japan in the year 2000. It is anticipated that the Study will be of interest to those concerned with the teaching of mathematics at the university level, to mathematics educators undertaking research in related areas, and to many other people with an interest in university level mathematics. The conference and publication related to this Study are likely to have a positive influence on the understanding and practice of the teaching and learning of mathematics at university level in the early years of the 21st century.

1. Why a study on the teaching and learning of mathematics at university level?

A number of changes have taken place in recent years which have profoundly affected the teaching of mathematics at the university level. Five changes which are still having considerable influence are

(i) the increase in the number of students who are now attending tertiary institutions;

(ii) major pedagogical and curriculum changes that have taken place at pre-university level;

(iii) the increasing differences between secondary and tertiary mathematics education regarding the purposes, goals, teaching approaches and methods;

(iv) the rapid development of technology; and

(v) demands on universities to be publicly accountable.

Of course, all of these changes are general and have had their influence on other disciplines. However, because of its pivotal position in education generally, and its compulsory nature for many students, it could be argued that these changes have had a greater influence on mathematics than perhaps on any other discipline.

There is no doubt that, in many countries, significantly more students are now entering university and taking mathematics courses than was the case ten years or so ago. On the other hand, an increasingly smaller percentage of students appears to be opting for studies which require substantial amounts of mathematics. Thus university departments are faced with a double challenge. On the one hand, they have to cope with the influx of students whose preparation, background knowledge and even attitudes are quite different to those of past students. On the other hand, they have to attract students to pursue studies in mathematics, where employment opportunities and well-paying jobs appear not to be as certain as in some other disciplines.

Some new developments in the teaching and learning of mathematics attempt to come to grips with these issues. For example, alternative approaches to calculus and linear algebra in the United States reflect, in part, attempts to make these subjects more engaging and meaningful for the majority of students. There have been content changes too, with increased emphases in some universities on applications and modelling, history and philosophy of mathematics, and so on. But a general perception remains in some quarters that the teaching of mathematics at the undergraduate level has not to date made sufficient effort to deal with the backgrounds and needs of present day students.

There is also often perceived to be a discontinuity between mathematics education in secondary schools and mathematics education in universities. Certainly the levels of ambition and demand placed on students are increased at the tertiary level. There is not the same attention paid to learning theories in the delivery of university mathematics as there is in the teaching of the subject at lower levels. University teaching methods tend to be more conservative. Often university teachers have joint responsibility for research and teaching. This is clearly beneficial but it can cause more emphasis to be placed on mathematical research in places where that is the main criterion for promotion.

Teachers of university mathematics courses, on the whole, have not been trained to, and do not often consider educational, didactic or pedagogical issues beyond the determination of the syllabus; few have been provided with incentives or encouragement to seek out the results of mathematics education. In days gone by responsibility was placed largely on students' shoulders: it was assumed that faculty's responsibilities were primarily to present material clearly, and that good students would pass and poor ones fail. The climate today is that academic staff are considered to have greater overall responsibility for students' learning. The role of instruction (specifically, of lectures) and staff accountability are being reconsidered.

Worldwide, increasing use is also being made of computers and calculators in mathematics instruction. Much mathematical software and many teaching packages are available for a range of curriculum topics. This, of course, raises such issues as what such software and packages offer to the teaching and learning of the subject, and what potential problems for understanding and reasoning they might generate. It would be good to collect examples of the use of information technology and software which enrich students' experience of mathematics and result in better understanding and learning.

Many academic mathematicians are aware of changes occurring around them, and of experimentation with different teaching approaches, but they have limited opportunities to embrace change owing to faculty structures and organisation. Further, the relationships between mathematicians in mathematics departments and their colleagues in mathematics education are often strained, with less productive dialogue between them than there might be. The same can be said of relationships between mathematicians and engineers, economists, etc., even though mathematics service teaching to students in other disciplines is an enormous enterprise. These general factors tend to work against, or delay, improvements in the teaching and learning of mathematics, particularly for those students whose main interests are in other disciplines.

As a result of the changing world scene, ICMI feels that there is a need to examine both the current and future states of the teaching and learning of mathematics at university level. The primary aim of this ICMI Study is therefore *to pave the way for improvements in the teaching and learning of mathematics at university level for all students.* To achieve this aim it is important for the professionals involved to

- exchange views and experiences from a wide variety of places and backgrounds;
- report about developments and projects that have taken place;
- consider the contributions from theory and research, and identify areas still to be investigated.

More specifically the Study will cover the following and related points:

◊ to identify, review, encourage, and disseminate research in educational matters at the tertiary level;

◊ to identify and describe major approaches to tertiary mathematics teaching within different cultures and traditions;

◊ to identify obstacles which might prevent the learning of mathematics;

◊ to discuss equity and other issues relating to mathematics education at university level;

◊ to discuss the goals of teaching mathematics to a range of students with different backgrounds and needs, and who should be responsible for that teaching;

◊ to find ways to meet changing needs without compromising the integrity of the subject;

◊ to identify, publicise, and expose to scrutiny, new teaching methods and the positive use of technology;

◊ to discuss the transition and the relations between secondary school and university;

◊ to consider ways to improve the preparation of teachers of mathematics at university level.

Leading up to and during the Conference relating to this Study, it is expected that there will be debate as to why mathematics is taught and what mathematics education is at university level. In addition, consideration will be given as to what is the current teaching and learning situation in universities, what it is believed that the situation should be, and how desired changes can be effected.

2. Themes and issues pertaining to research on the teaching and learning of mathematics at university level

Most academic mathematicians know little about the research that has been undertaken in mathematics education in general, or at the tertiary level in particular. Generally speaking, they are unaware of the methods used by researchers in education. One of the most valuable aspects of the current study is that it could collect together the major findings of mathematics education research, review them, and make them readily accessible to a wide audience. The potential usefulness and limitations of this research should then be considered in the light of the practice of teaching. At the same time, it would be valuable to determine research areas which have not yet been explored and to encourage work in them.

The following questions are of particular interest for the Study.

※ *What is mathematical understanding and learning, and how are these achieved? What are the underlying theories behind these and how do they relate to teaching at university level?*

※ *What research methods are employed in mathematics education? What are the major research findings of mathematics education? What are the obstacles to having teaching practice become informed/influenced by research findings?*

※ *Might insights into the nature of the learning process play out differently at different grade levels? Are the theories that are relevant at school level, relevant at university level as well? Is there a need for theories that are specific to university level?*

※ *What research has there been into traditional and alternative methods of teaching and what do the results of such research tell us?*

※ *In what ways can teaching change to take into account the different background, abilities and interests of the learner? What methods are effective for teaching large classes?*

※ *What do we know about the learning and teaching of specific topics such as calculus and linear algebra? Are there characteristics which are relevant to specific topics? Are there characteristics which are pertinent to a number of topics?*

※ *What alternative forms of assessment exist? How can assessment be used to promote better learning and understanding?*

- *What are the mathematical competences that are required in the different professions ?*
- *What are students' attitudes and beliefs concerning mathematics ? What causes them to change ? How do these affect their enrolments and success in courses with substantial mathematical components ?*
- *What are the effects of the use of technology in the teaching and learning of mathematics ? In what ways can technology be used to enhance understanding ?*
- *What important issues are under-represented in the research literature and how can researchers be encouraged to work in these areas ?*

3. Themes and issues pertaining to practice

We divide this section into four parts: Clientele, Curriculum, Student activity, and Pedagogy.

Clientele

The students who are of interest for this Study include all those students who are taught mathematics at university level, whether as mathematics majors, as students of other subjects using mathematics as a service course, as prospective mathematics teachers, or as recipients of some form of general mathematics appreciation course. Hence we are addressing the needs of not only future research mathematicians but also other categories of future mathematics professionals as well as graduates in other disciplines who require varying amounts of mathematical knowledge, skill or insight.

For several reasons, in many countries there has been a move to mass education at university level. As a result many mathematics departments are providing courses for a much wider range of ability and needs than was formerly the case. Simultaneously with this increase in student numbers, there has been a change in the kind of student preparation in secondary schools as well as in students' interests and motivation. Consequently many students have not met material which was in most secondary school curricula of the 1970s. In addition they may have been taught by an approach which places more emphasis on the intuitive and pragmatic. Some university mathematics departments have been slow in recognising these changes in their student intake. Others have developed new courses to cope for the range of content needs but have made few pedagogical concessions.

There are a number of special groups of students including potential teachers of school mathematics, scientists, engineers. What should the interaction between mathematical and professional knowledge be ? To what extent do these groups need specially designed courses ?

Curriculum

By curriculum we mean matters pertaining to the purposes, goals and content of mathematics education. Current curricula may need to be reconsidered for at least two reasons. There are the different student needs that were mentioned above and there are the developments in mathematics itself.

As far as the changing clientele is concerned, it is not clear that its constitution or its needs have been adequately considered. What are the professional aspirations of our student population ? Will they go on to be teachers, to work in industry, to be academics, etc. ? How should the curriculum be shaped to meet the needs of these groups ?

What changes are, or should be, taking place in the curriculum ? Some mathematical subject areas are on the decline while others are in the ascendancy. What is the rationale for the changes ? Are some content areas now less important and should other areas take their place ?

Mathematics as a rapidly developing research field is continuously undergoing changes with new fields arising, changes of emphasis, and so on. At present we notice strong interactions between different branches, an increasing interest in applications, the development of an experimental approach, etc. To what extent is and should this evolution be reflected in the teaching of the subject at undergraduate level ?

Student activity

Here we wish to discuss the various ways in which students might be induced to interact with mathematical content, both inside and outside the classroom. What forms of study and what activities are currently used in the teaching of mathematics ? Do different forms of engagement (e.g., in “mathematics labs” where students explore families of mathematical objects using computers) have the potential to result in better learning in different subjects ?

Two of the central issues here are the role of the student and the attitude towards the subject. Under what circumstances should the student’s role be to receive information and when should it be to interact with the content in more dynamic ways (including exchanges with their teachers and with other students) ? Under what circumstances should the subject be presented as a set of skills (algorithms), as a set of processes or as a combination of these ? The attitude of the teacher will require different reactions and actions from students.

Pedagogy

By pedagogy we mean the teachers’ orchestration of teaching and learning environments and situations, examined both from the descriptive/analytic position (what *is* the case ?) and the normative position (what *ought to be* the case ?).

Some areas of mathematics are met by students before they enter university and the approaches they have met in school may well be quite different from those which are common in universities. Mathematics majors, for example, have to meet a more formal approach to calculus/analysis. What are the best ways to effect this change of approach ? But, given the changes in clientele referred to earlier, it is likely that the transition to university teaching poses problems for all students. How can the transition from school to university be best accomplished ?

This raises the issue of the philosophical approach to the subject. Many courses appear to concentrate on content knowledge. The emphasis seems to be on learning certain algorithms or theorems and applying them in controlled situations. This hides the creative and problem solving aspects of the subject. Should more emphasis be placed on the way that mathematicians think and create ? Should there be more emphasis on students’ problem solving capabilities as opposed to their learning the results the subject produces ? How can the impact of problem-based lectures, the use of computers, project work and so on, be assessed ?

One of the issues that requires discussion is the importance placed upon teaching by universities generally. In many universities, promotion is based largely on research output, with teaching having a minor role. In such places, there is little incentive for academics to put more emphasis on their teaching. There are, of course, many academics who put quite a lot of work into their teaching. Should the profession, through its national bodies, show that it recognises the importance of teaching at the university level ?

Another relevant issue is, where and how do academics learn to teach ? Some universities have courses for their staff but these often do not go into any great depth in particular subject areas. Should more formal instruction be given and, if so, by whom and of what type ?

Now that there is relatively ready access to computers, graphical calculators and calculators, it is worth examining to what extent we can release our students from some of the drudgery experienced by past generations. How has the new technology changed the content and philosophy of the curriculum ? How can mathematics majors benefit from using computer technology ? How can majors in other subjects benefit ? Should existing programmes be delivered in the same way as in the past or can technology assist in the development of higher order skills or other more important skills ?

4. Themes and issues relating to policy

Policy issues naturally fall into two groups: those relating to society at large and those which are the concern of a specific university or university department.

Society

The amount of control that society, through its government, takes over its universities, varies considerably from country to country. In most countries, government provides the majority of the financial support for its universities. Hence, at least indirectly, government policies will affect individual departments. How are these policies formed ? What influence can and should mathematicians and mathematics educators have on them ?

The previously mentioned increasing number of students at the university level has, in many nations, occurred either explicitly or implicitly as the result of government policy. Is there cause for satisfaction with the result of this policy or is there a need to change or modify it in some way ?

The mathematical community is convinced of the importance of mathematics both for its own sake and for the contribution that it ultimately makes to society. It is not clear that society in general also holds this position. Perhaps it does not realise what it takes to generate the contribution mathematics can make. What does the mathematical community need to do to make society aware of the mathematical requirements of society and how these can be achieved ? What does the mathematical community need to do to make mathematics more visible in a competitive environment ? In what ways should society provide its citizens with the basic ideas and philosophy of mathematics and its impact on our lives, both from a philosophical and practical point of view ?

UNIVERSITY

In some countries the difference between universities and other tertiary institutions is the fact that research takes place in universities. In such countries, universities have a research culture in which it is assumed that most lecturers will engage in research. To what extent should the teaching of mathematics be delivered by lecturers who are engaged in some form of research ?

In some countries, university degrees are of a general nature and cover a range of topics. In other countries, there are more directed programmes for students to follow. What is more, some of the more applicable areas of mathematics may be taught outside a mathematics department by engineers, statisticians, physicists, etc. To what extent should courses be general and to what extent should they be specific to each user group ? To what extent should courses be taught by mathematicians and to what extent should they be taught by experts from other appropriate fields ?

What then is the role of a department of mathematics at the end of the twentieth century, given that there is a tendency for non-mathematics departments to teach their own mathematics ? (This is not only for bureaucratic reasons but also because these departments are often dissatisfied with the gap between the content and approach they require and the content and approach of mathematics departments.) Should departments of mathematics be responsible for *all* of the students taking mathematics at its university or should it concentrate on its traditional clientele, the mathematics majors ? Will departments which do not teach a range of students remain viable in an environment where a balanced budget, rather than education, is the main concern of administrators ? What cooperation can there be with other disciplines for whom mathematics is a service course ? In some cases there is an overlap in the material being taught in courses by a mathematics department and a service department. Are there good reasons for continuing this practice ?

Clearly no university department can teach all branches of mathematics. Are there fundamental branches of the subject which should be in all programmes ? How should the balance be struck between suitable major components ?

How strongly are incoming students influenced by career prospects in mathematics ? How should this affect the courses offered and the advice given to prospective students ?

5. CALL FOR REACTIONS

The work of this Study will take place in two parts. The first consists of a conference which is to be held in Singapore from December 8 to 12, 1998. *English will be the language of the conference.* The conference will be a working one, where every participant will be expected to be active. Current planning is for a limited attendance of about 75 persons.

Given the style of the conference, we anticipate a variety of types of contributions that will be presented in plenary sessions, working groups, panels and short presentations. Presentations may include position papers, discussion papers, surveys of relevant areas, reports of projects, or research papers of an educational nature.

We invite contributors to make a submission for consideration by the International Programme Committee no later than 1 May 1998. Submissions should be up to three pages in length and may be e-mailed, faxed or sent as hard copy. They should be related to the problems and issues identified in this document but need not be limited to these alone. One might also draw to the attention of the Committee the names of other people whom one feels ought to be invited, stating the type of the contribution they might make. We would appreciate knowing the nature and results of related studies in this area.

Participation in the conference is by invitation only. Invitations to those whose submissions have been accepted will be made in July 1998. At the same time invitees will be asked to produce a longer version of their submission for publication in the pre-conference proceedings. The Study organisers are seeking funds to provide partial support to enable participants from non-affluent countries to attend the conference but it is unlikely that full support will be available for any one individual.

All contributions and suggestions concerning the content of the study and the conference programme should be sent to

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The second part of the Study is a publication which will appear in the *ICMI Study Series*. This publication will be based both on the contributions requested above and the outcomes of the conference working group and panel deliberations. The exact format of the publication has not yet been decided but it is expected to be an edited, coherent book which it is hoped will be a standard reference in this field for some time.

The planned timetable for the Study is as follows:

1 MAY 1998:
Deadline for worldwide reaction to this Discussion Document.

1 JULY 1998:
The Study conference programme and the list of invitees to be finalised.

8–12 DECEMBER 1998:
Study conference, Singapore.

1 MARCH 1999:
Deadline for the submission of papers to the study publication.

31 JULY–7 AUGUST 2000*:
Presentation of main considerations and findings, ICME–9, Makuhari, Japan.

1999–2001:
The editors produce the study volume.

(* Actual dates to be confirmed.)

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