4. Geometry as reflected in education

need for more teachers has caused, on average, a decline in their university preparation, especially with respect to the more demanding parts of mathematics, in particular geometry. Since younger teachers have learned mathematics under curricula that neglected geometry, they lack a good background in this field, which in turn fosters in them the tendency to neglect the teaching of geometry to their pupils.

The situation is even more dramatic in those countries which lack a prior tradition in schooling. In some cases geometry is completely absent from their mathematics curricula.

— The gap between the conception of geometry as a research area and as a subject to be taught in schools seems to be increasing; but so far no consensus has been found on how to bridge this gap, nor even whether it could (or should) be bridged through an introduction of more advanced topics in school curricula at lower grades.

4. Geometry as reflected in education

In former sections, we have considered geometry mainly as a mathematical theory and have analyzed some aspects of its *teaching*. Since *learning* is unquestionably the other essential pole of any educational project, it is now appropriate to pay due attention to the main variables which may affect a coherent teaching/learning process. Consequently, several different aspects or "dimensions" (considered in their broadest meaning) must be taken into account:

— *The social dimension,* with two poles:

- The cultural pole, i.e. the construction of a common background (knowledge and language) for all the people sharing a common civilization;
- The educational pole, i.e. the development of criteria, internal to each individual, for self consistency and responsibility.

— *The cognitive dimension,* i.e. the process which, starting from reality, leads gradually to a refined perception of space.

— *The epistemological dimension*, i.e. the ability to exploit the interplay between reality and theory through modelling (make previsions, evaluate their effects, reconsider choices). Thereby axiomatization enables one to get free from reality; this in turn may be seen as a side-step which allows further conceptualization.

— *The didactic dimension,* i.e. the relation between teaching and learning. Within this dimension several aspects deserve consideration. As an example, we list three of them:

- To make various fields interact (both within mathematics and between mathematics and other sciences).
- To make sure that the viewpoints of the teacher and the pupils are consistent in a given study. For instance, to be aware that different distance scales may involve different conceptions and processes adopted by the pupils, even though the mathematical situation is the same: in a "space of small objects", visual perception may help to make conjectures and to identify geometric properties; when dealing with the space where we are used to move around (the classroom, for instance) it is still easy to get local information, but it may be difficult to achieve an overall view; in a "large scale space" (as is the case in geography or in astronomy) symbolic representations are needed in order to analyze its properties.
- To pay due consideration to the influence of tools available in teaching/learning situations (from straightedge and compass, as well as other concrete materials, to graphic calculators, computers and specific software).

It goes without saying that all these dimensions are interrelated with each other and that they should also be related appropriately to different age levels and school types: pre-primary level, primary level, lower secondary level, upper secondary level (where differentiation into academic, technical, vocational tracks usually starts), tertiary (i.e. university) level, including teacher preparation.

5. New technology and teaching aids for geometry

There is a long tradition of mathematicians making use of technological tools, and conversely the use of these tools has given rise to many challenging mathematical problems (e.g. straightedge and compass for geometric constructions, logarithms and mechanical instruments for numerical computations). In recent years new technology, and in particular computers, has affected dramatically all aspects of our society. Many traditional activities have become obsolete, while new professions and new challenges arise. For instance, technical drawing is no longer done by hand. Nowadays, instead, one uses commercial software, plotters and other technological devices. CAD/CAM and symbolic algebra software are becoming widely available.