Thus – roughly speaking – for fixed T the eigenvalues λ_n are "proportional" to the coefficients $a(nT)$.

Suppose that F is in $S_k^*(\Gamma_2)$. Using Theorem 1 in §2 and the estimate (8) with $m = 1$ one finds that

$$a(T) \ll_{\varepsilon, F} (\det T)^{k/2 - 1/2 + \varepsilon} \qquad (\varepsilon > 0)\,, \tag{15}$$

and (11) together with (14) implies that (15), in fact, is best possible.

On the other hand, taking into account (12) and the fact that the Hecke eigenforms form a basis, one may be led to the following

CONJECTURE 1 [11]. *Let F be a cusp form of integral weight k on Γ_2 and suppose that either k is odd or that k is even and F is in the orthogonal complement of the Maass space. Let $a(T)$ (T a positive definite symmetric half-integral* (2, 2)*-matrix) be the Fourier coefficients of F. Then*

$$a(T) \ll_{\varepsilon, F} (\det T)^{k/2 - 3/4 + \varepsilon} \qquad (\varepsilon > 0)\,.$$

Concerning norms of Fourier-Jacobi coefficients, one optimistically may hope for the truth of the following

CONJECTURE 2. *Let F be a cusp form of integral weight k on Γ_2 and denote by ϕ_m ($m \in \mathbf{N}$) its Fourier-Jacobi coefficients. Then*

$$\| \phi_m \| \ll_{\varepsilon, F} m^{k/2 - 1/2 + \varepsilon} \qquad (\varepsilon > 0)\,. \tag{16}$$

Note that – in view of Theorem 1 of §3 – (17) would be best possible.

NOTE ADDED IN PROOF

The estimates for the Fourier coefficients of cusp forms of arbitrary genus $n \geqslant 2$ obtained in [7] improve upon those obtained in [5, 11], cf. 4.2. ii).

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