1. Introduction

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SMOOTHLY EMBEDDED 2-SPHERES AND EXOTIC 4-MANIFOLDS

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1. INTRODUCTION

We prove two independent results concerning multiple smooth structures on certain topological 4-manifolds with boundary (theorems 1.1 and 1.2). The second of our results has been already proven by Akbulut [A]; we give here a different proof. The common underlying theme consists of obstructions to represent 2-homology classes of 4-manifolds by smoothly embedded spheres. Indeed, we use the fact that certain classes of self-intersection -1 or -2 cannot be represented by smoothly embedded spheres inside certain 4-manifolds, while they can inside others.

With our first result we give a partial answer to the following question, raised in [G]: given the 4-manifold with boundary $M(k, l, m)$, obtained by attaching 2-handles to B^4 according to the framed link of figure 1, how many diffeomorphism types are realized by permuting the integers k, l, and m?

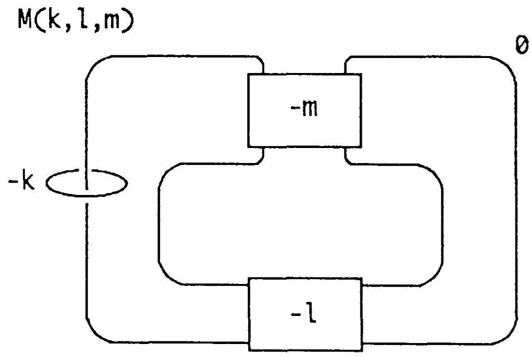

FIGURE 1

(In such pictures, a box with $-m$ inside means $-m$ full twists. A $-k$ next to a loop means that the francing has linking number $-k$ with the loop.)

Notice the equality $M(k, l, m) = M(k, m, l)$, obtained by an obvious isotopy of the link. In section 2 we prove the following:

THEOREM 1.1. *Let* $k, l, m, k \neq l$, *be positive integers with either* $k \leqslant 2$ *or* $l \leqslant 2$, *and* s *a non-negative integer. If either* (i) $s \neq 0$ *or* (ii) $s = 0$ *and* $k \equiv l \bmod 2$, *then the manifolds* $M(k, l, m) \overset{s}{\#} \overline{\mathbf{CP}}^2$ *and* $M(l, k, m) \overset{s}{\#} \overline{\mathbf{CP}}^2$ *are homeomorphic but their interiors are not diffeomorphic.*

We have as a corollary that for n odd $M(1, n, 1)$ and $M(n, 1, 1)$ are homeomorphic but not diffeomorphic, a result already obtained in [G] by a different method.

As for the second result, let W_i, $i = 1, 2$ be the 4-manifolds with boundary obtained via the framed knots K_i, $i = 1, 2$ of figure 2.

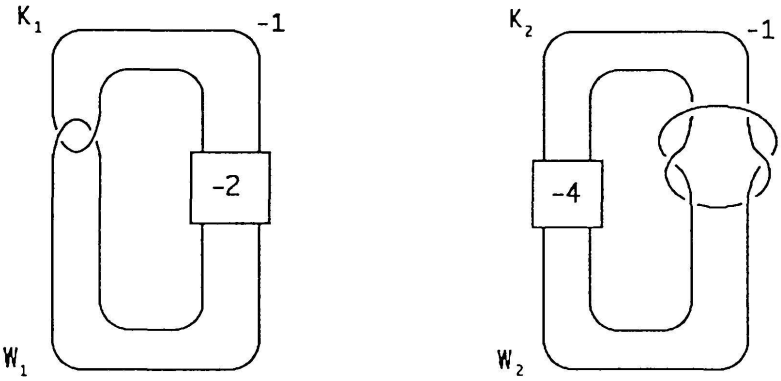

FIGURE 2

We give a new proof of the following theorem, due to Akbulut [A]:

THEOREM 1.2. W_1 *and* W_2 *are homeomorphic but their interiors are not diffeomorphic.*

Remark. In Akbulut's notation, the manifolds W_1 and W_2 are respectively $-Q_1$ and $-Q_2$.

To prove theorems 1.1 and 1.2 we use gauge-theoretical results about the smooth structure of blown-up elliptic surfaces [FM2]. We remark that also the analogous results from [G] and [A] depend on gauge theory.

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