# IV. A Skein Model for the Kauffman polynomial

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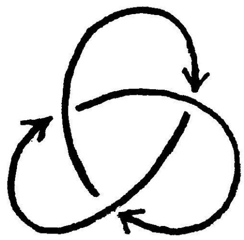
is admissible and contributes $(-z)^3\delta$ to the summation for the negative trefoil



## IV. A Skein Model for the Kauffman polynomial

The work of section 3 goes over essentially verbatim for the Dubrovnik version of the Kauffman polynomial. Recall that in this context a template is obtained by first orienting the edges of the universe $U$ underlying the unoriented $K$, and then labelling the edges of $U$.

The chart in Figure 7 shows the cases of admissible splices at crossings (with respect to the skein template algorithm). Each splice has been labelled with its corresponding vertex weight. Note that a splice is admissible if it indicates the form of passage that is obtained from an approach to the crossing that meets it as an under-crossing. Such approaches give active crossings in the skein template algorithm.

I have retained only the arrow for the first passage after each split, because the orientation on the other edge may change under the direction of the template. *The crossings are oriented because each end-node (unlink) produced by the skein template algorithm acquires an orientation from the directions of travel given by the template.*

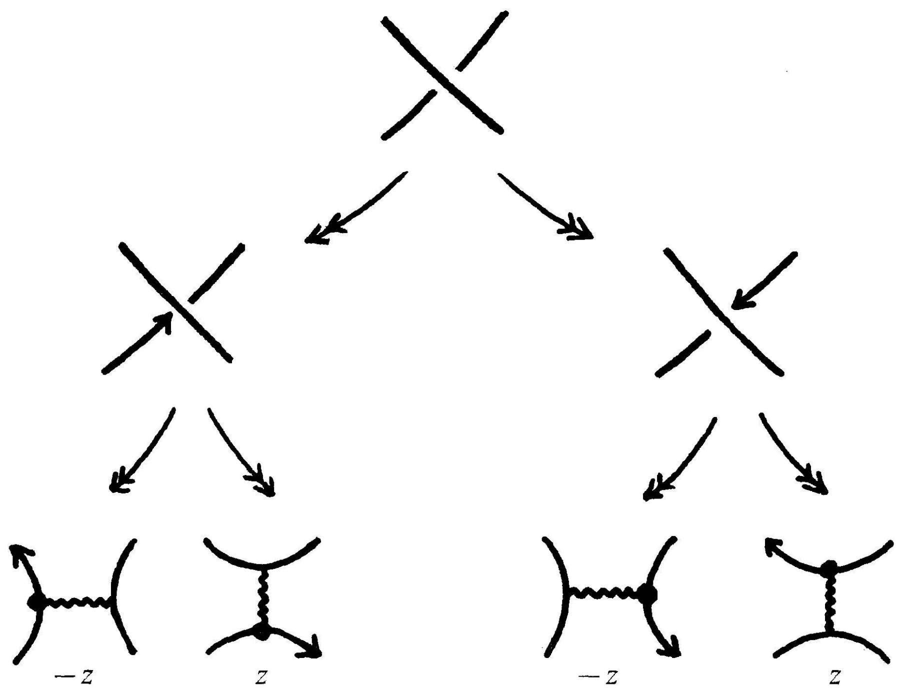
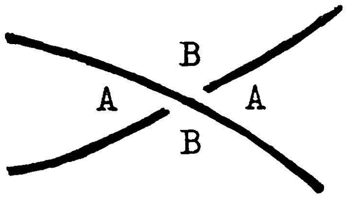



Figure 7

In order to understand the pattern of these admissible splices, consider an unoriented crossing




I have labelled two of its regions $A$. These are swept out by turning the overcrossing line counterclockwise. The other two regions are labelled $B$. The *$A$-splice* of the crossing is that splice that joins the two regions labelled $A$. The *$B$-splice* joins the two regions labelled $B$.

We then see that a passage is admissible in the $A$-splice if it occurs on the *right* for an observer who stands in between the strands, facing in the direction of the passage from basepoint. Similarly, the admissible $B$-splices are on the left for such an observer.

Call an admissible splice *negative* if it is of $B$-type. (This receives a $(-z)$ in Figure 7.)

With these definitions we have

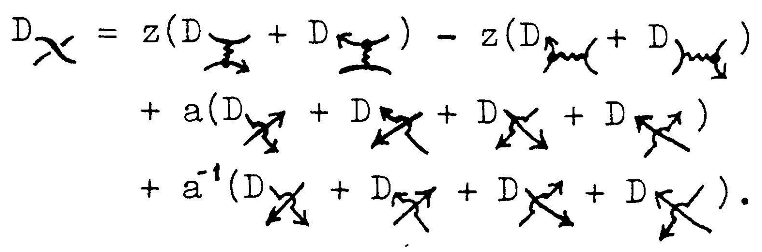
$$t(L) = \text{number of splices to obtain } L \text{ from } K\,.$$
$$t_-(L) = \text{number of negative splices}\,.$$

$\mathrm{Au}(K, T)$ = set of admissible unlinks relative to $K$ and $T$ (here $K$ is unoriented).

Then the Dubrovnik polynomial is given by the formula

$$D_K = \sum_{L \in \mathrm{Au}(K,T)} (-1)^{t_-(L)} z^{t(L)} a^{w(L)} \mu^{|L|-1}$$
$$(\mu = 1 + (a - a^{-1})/z)\,.$$

As a state-expansion we can write



Once again,

$$D_K = \sum_{L \in \mathrm{Au}(K,T)} <K \mid L> \mu^{|L|-1}$$

where $<K \mid L>$ denotes the product of vertex weights (all relative to the given template). Independence of the template follows from the well-definedness of the polynomial itself.

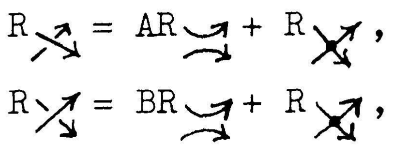
*Remark.* It would be very interesting to know the relationship between this state model for the Kauffman polynomial and the extraordinary model of Jaeger [34]. Jaeger gives a state expansion where the states are a collection of oriented knots and links. Each state is itself evaluated via the regular isotopy version of the Homfly polynomial.

## V. Graph polynomials

The two skein polynomials (Homfly and Kauffman) each have three variable extensions to rigid vertex isotopy invariants of 4-valent graphs imbedded in three-space. This construction has been announced in [45]. (See also [56] and [74].) Our skein models involve 4-valent graphs implicitly, and so give rise to a natural definition for these extended polynomials as state models.

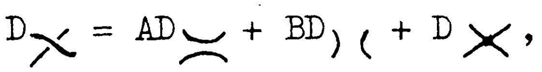
Let the new variables $A$ and $B$ be given, with $z = A - B$ the usual $z$ for the skein polynomials. The extended polynomials are then defined by the axioms:

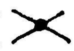
### Homfly Extension Axioms

1. 

2. $R_K$ = usual regular isotopy Homfly polynomial if K is free of graphical vertices ().

### Kauffman Extension Axioms

1. 

2. $D_K$ = usual regular isotopy Dubrovnik polynomial if K is free of graphical vertices ().