4. CONCLUDING REMARKS

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Clearly for $h = u(f, gi(i=1, \dots k(f))) = u(f, g)$:

$$\sum_{i=1,\dots k(f)} w(Di_{gi,1}) = w_o(D, h), \sum_{i=1,\dots k(f)} w(Di_{gi,2}) = w_e(D, h)$$

and

$$\sum_{i=1,\dots k(f)} r(Di_{gi,2}) = r_e(D, h).$$

Moreover, since h is easily seen to be compatible with f and the gi, it follows from the Unification Lemma that $<D \mid f> \prod_{i=1,\dots k(f)} <Di \mid gi> = <D \mid h>$. Hence

$$<D \mid f> \prod_{i=1,\dots k(f)} A(Di, z) =$$
$$t^{r(D)-k(f)} \sum_{g \in T(D,f),\, h=u(f,g)} <D \mid h> t^{w_o(D,h)} (-t)^{-w_e(D,h)} (-1)^{r_e(D,h)}$$

It now follows from Proposition 9 that

$$H(D, z, a) = \sum_{f \in CL(D)} \sum_{g \in T(D,f),\, h=u(f,g)}$$
$$<D \mid h> (t^{-1}a)^{-r(D)+k(f)} ((a-a^{-1})z^{-1})^{k(f)-1} t^{w_o(D,h)} (-t)^{-w_e(D,h)} (-1)^{r_e(D,h)}$$

In this expression $k(f)$ can be replaced by $k'(h)$. Moreover u clearly defines a bijection from the set $\{(f, g)/f \in CL(D),\ g \in T(D, f)\}$ to $C'L(D)$. This completes the proof.

4. Concluding remarks

1. It would be interesting to generalize the results of Section 3 to arbitrary diagrams. This is done for Proposition 6 in a joint paper with Louis Kauffman (in preparation).

(2) Since the topological and algebraic aspects of the Alexander polynomial are well understood, one may try to use Proposition 9 to gain some insight of the same kind on the homfly polynomial. Clearly one can combine Proposition 9 with classical results which relate the polynomial $A(D, z)$ to Burau matrices, Seifert surfaces and matrices, presentations of the fundamental group of the complement... This leads to corresponding complex labelled structures which seem to be worth studying. As for the combinatorial aspects Proposition 12 is only a first step and some further progress closely connected with Proposition 9 is reported in the following forthcoming papers: [A] Circuit partitions and the homfly polynomial of closed braids, Trans. AMS, to appear, [B] A combinatorial model for the homfly polynomial, preprint.

(3) We believe that our approach to the Morton-Franks-Williams inequalities could be developed to yield more precise results on the cases of equality (which appear to be extremely frequent). For instance it is shown in [A], Proposition 7 that the first (respectively: third) inequality of Proposition 10 holds with equality if all vertices of D have negative (respectively: positive) sign.

(4) Proposition 12 can be interpreted in terms of explicit matrix representations of Hecke algebras. The homfly polynomial of a braid diagram appears as the trace of a matrix indexed by its labellings. This matrix can be computed as a product consisting of one matrix for each crossing (which incorporates the interaction and writhe contributions) and a final diagonal matrix which assigns suitable weights to the various labellings.

(5) The known relationship of the Jones sequence of state models with solutions of the Yang-Baxter equation suggests that theoretical physics might provide an interpretation of the properties of the product operation for homfly polynomials. An algebraic generalization of this operation is given by V. G. Turaev in: Algebras of loops on surfaces, algebras of knots, and quantization, LOMI preprint E-10-88.

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