2. Lifting edges

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quotient graph X/G. When G acts on X we shall often say that G is a *group of automorphisms of* X.

We adopt the usual notation whereby G_x denotes the stabilizer of a vertex x. If $g \in G$ happens to fix x we write g_x for the element g thought of as a member of G_x. Of course G_e denotes the stabilizer of the edge e. If x is a vertex of e then G_e is a subgroup of G_x.

Suppose G acts on a tree X. If $g \in G$ fixes the vertices u, v then it must fix the whole geodesic $\overrightarrow{uv}$, since otherwise the image of $\overrightarrow{uv}$ under g would be a second geodesic from u to v.

2. Lifting edges

Let G be a group of automorphisms of a tree X. Choose a maximal tree M in X/G and lift it [4, Proposition I.14] to a subtree T of X. The vertices of T form a set of representatives for the action of G on the vertices of X. For each pair of edges f, $\bar{f}$ from $X/G - M$ select one, say f, and lift it to an edge e of X which has its initial vertex x in T. Exactly one vertex z of T lies in the same orbit as $t(e)$ and we choose an element γ_f from G that maps z onto $t(e)$. We can now lift $\bar{f}$ to $(\gamma_f)^{-1}\bar{e}$. This has its initial vertex z in T and $\gamma_{\bar{f}} = (\gamma_f)^{-1}$ sends the vertex x of T to its terminal vertex (Figure 1). Finally we extend the correspondence $f \to \gamma_f$ over the edges of M by setting $\gamma_f = 1$ (the identity element of G) whenever $f \in M$.

The *Bass-Serre theorem* [4, Theorem I.13] gives the following presentation for G.

(a) *Generators.* The elements of all the G_w where w is a vertex of T and the γ_f where f is an edge of X/G.

(b) *Relations.* The internal relations of each stabilizer G_w together with

$$\gamma_f = 1 \text{ if } f \text{ is an edge of } M,$$

$$\gamma_{\bar{f}} = (\gamma_f)^{-1} \text{ and}$$

$$\gamma_{\bar{f}}\, g_x \gamma_f = (\gamma_{\bar{f}}\, g\, \gamma_f)_z \text{ where } e \text{ is the chosen lift of } f \text{ and } g \in G_e.$$

(If f is an edge of M then $z = t(e)$ and the final relation reduces to $g_x = g_z$ whenever $g \in G_e$).