Note (Added in proof)

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We now define the functions h_j as before by

$$h_j(k) = \begin{cases} a_{kj} & \text{for } j \leqslant k \\ h_{j-1}^2(k) & \text{for } j > k\,. \end{cases}$$

A simple induction argument now shows that (1) holds for the axioms and hence that $\mathscr{F}/D$ is a model of Peano arithmetic. If as in Theorem 3 the a_{kk} is chosen as the least number so that the sequence $a_{k1}, \ldots, a_{kk}$ satisfies the above conditions then a true statement which is false in $\mathscr{F}/D$ may be constructed via the method of Theorem 3.

The disadvantage of this direct approach is that the model $\mathscr{F}/D$ constructed in this manner is dependent in its definition upon logical formulas and so is not as purely an algebraic construction. Moreover the independent statement which results has no simple combinatorial expression as have those given in Sections IV, V, and VI. Note that in this approach we have not used the property peculiar to Peano's axioms concerning the limited associates of the axioms which is expressed in the proof of Theorem 4. This shows that the method outlined here applies to any recursively enumerable set of axioms for arithmetic which is sufficient to allow the coding required for Theorem 3. Thus, we may prove a general form of Gödel's Incompleteness Theorem without the use of self-reference techniques. At the same time the very generality of the approach outlined here indicates that there is no hope by this method to avoid the use of metamathematics. It is only the above-mentioned property of the Peano axioms vis-à-vis limited formulas that allowed us the latitude to define suitable functions h_j, and hence the model $\mathscr{F}/D$, by means of a combinatorial principal without reference to logical formulas.

Note (Added in proof)

The first sentence of the section entitled "Added in proof" of Kochen and Kripke [12] p. 294, which was inserted by the second author, is not correct and should be deleted in favor of the following corrected version. The first author proposed that the Paris-Harrington statement is false in an initial segment of any non-standard model, and this was verified jointly by the two authors. Adapting this idea, the second author defined the set $\mathscr{F}$ of functions which result in the model of Section V. The first author subsequently found the new set $\mathscr{F}$ of functions which define the simpler model of Section VI.

The devices used in Section III are an adaptation of the ideas in Paris-Harrington [3].

The approach outlined in (d) of Section VII is due to the second author and leads to a concept of 'satisfying' formulas by finite sequences called *fulfillability* wich leads to model-theoretic proofs of many theorems (such as Gödel's and Rosser's theorems) usually proved proof-theoretically and to other applications to the model theory and proof theory of arithmetic. It will be developed in a subsequent paper of the second author.

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