

# 10. Transfer

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$$E_2^{pq} \cong H^p(Q, H^q(K, M)) \quad (2)$$

converging to the graded group associated with a filtration of the cohomology  $E^{p+q}(G, M)$ . In (2), the cohomology  $H^q(K, M)$  of the subgroup  $K$  is suitably interpreted as a  $Q$ -module, so that the outside cohomology is defined. The essential portions of such a spectral sequence were discovered by R. Lyndon in his 1946 Harvard thesis, at about the same time that Leray was formulating the general notion of a spectral sequence. Lyndon did use his formulation for computation. Some years later [1953], Hochschild and Serre formulated a spectral sequence like that of (2) in the conventional language, so such a sequence is usually called a Hochschild-Serre spectral sequence. (There are actually several different constructions of such a sequence, and some residual uncertainty as to whether these constructions all yield the same spectral sequence). The essential observation is that computing cohomology or homology in a fiber situation like that of (1) inevitably leads to the spectral sequence technology—whether the fiber situation is group theoretic, as with the exact sequence (1), or a fiber space, as in the case so effectively exploited by Serre in topology.

## 10. TRANSFER

The operation of *transfer* was well known in group theory, beginning with Burnside's work on monomial representations. If  $H$  is a subgroup of index  $n$  in  $G$ , the transfer from  $G$  to  $H$  is a homomorphism.

$$t : G/[G, G] \rightarrow H/[H, H] \quad (1)$$

between the factor-commutator groups. To define it, choose representatives  $x_1, \dots, x_n$  of the right cosets of  $H$  in  $G$ , so that  $G = \cup Hx_i$  and write  $\rho(x)$  for the representative  $x_i$  of the coset  $Hx$ . Then  $t$  is

$$t(g) = \prod_{i=1}^n (x_i g) [\rho(x_i g)]^{-1} \quad (2)$$

This map  $t$  is independent of the choice of the set of representatives  $x_1, \dots, x_n$ .

Since the factor commutator group  $G/[G, G]$  in (1) is simply the 1-dimensional homology group  $H_1(G, \mathbf{Z})$ , the transfer can be regarded as a map in homology.

$$t : H_1(G, \mathbf{Z}) \rightarrow H_1(H, \mathbf{Z})$$

In 1953 Eckmann extended this map to apply in all dimensions, both in homology and cohomology. Using the standard homogeneous complexes

$B(G)$  and  $B(H)$  for the groups  $G$  and  $H$ , he defined a cochain transformation  $t$  for any  $G$ -module  $A$  and any cochain  $f$  by

$$(tf)(g_0, \dots, g_p) = \sum_{i=1}^n x_j^{-1} f(x_j g_0 (\rho(x_j g_0))^{-1}, \dots, x_j g_n (\rho(x_j g_n)) - 1)$$

This map, up to chain homology, is again independent of the choice of the representatives  $x_j$ , so yields a homomorphism

$$t: H^p(H, A) \rightarrow H^p(G, A).$$

On the other hand, each cochain of  $G$  over  $A$  automatically restricts to a cochain of  $H$  over  $A$ ; this process defines the *restriction* map

$$r: H^p(G, A) \rightarrow H^p(H, A).$$

Eckmann proved that the composite  $tr$  of these maps is the endomorphism given by multiplication by  $n$  in  $H^p(G, A)$ : He made a variety of applications. The notion of transfer was also used by Artin and Tate (see below) in class field theory.

The discovery of the homology of a group had the feature that it exhibited a “non-obvious” construction on groups; in much the same way, the discovery of transfer produced a non-obvious homomorphism between cohomology groups. Thus it is that recently Kahn and Priddy have been able to construct the transfer homeomorphism for the generalized cohomology of an  $n$ -fold covering  $\Pi: E \rightarrow B$ . This transfer applies to the cohomology with coefficients in any strict  $\Omega$ -spectrum; when applied to the Eilenberg-Mac Lane spectrum  $K(\Pi, n)$ , the generalized cohomology is ordinary cohomology and the transfer agrees with the classical one. Using this transfer, they prove a conjecture of Mahowald and Whitehead about a “canonical map” of the  $n$ -fold suspension  $\Sigma^n \mathbf{R}P^{n-1}$  of the real projective  $n - 1$  space into the  $n$  sphere. This map  $\lambda$  is the adjoint of the map

$$\mathbf{R}P^{n-1} \rightarrow O_n \rightarrow \Omega^n S^n.$$

Here the first arrow takes a line through the origin in  $\mathbf{R}^n$  into the reflection in the plane perpendicular to that line; while the second arrow represents each element of  $O^n$  as a map of  $(\mathbf{R}_n \cup \infty, \infty)$  into itself, and hence as an element of the  $n^{\text{th}}$  loop space of  $S^n$ .

The result of Kan and Priddy is that  $\lambda$  is an epimorphism of 2-primary components in stable homotopy.