Because of the assumed dominated variation $M\{(x, \infty)\}$ decreases more slowly than a certain power $x^{-\alpha}$, and hence the quantity (7.9) is o (M $\{(ax, \infty)\}$ for any fixed $a>0$. Since $Z \leqslant 0$ and X has a distribution of the form (7.3) we conclude that

$$(7.10) \qquad P\{X+Y+Z>x\} \leqslant P\{X>(1-\varepsilon)x\}+P\{Y>\varepsilon x\} \sim \\ \sim M\{((1-\varepsilon)x, \infty)\}.$$

On the other hand,

$$(7.11) \qquad P\{X+Y+Z>x\} \geqslant P\{X>(1+\varepsilon)x\} \cdot P\{Y+Z>-\varepsilon x\},$$

and the last factor tends to 1 as $x \to \infty$. The probabilities on the left are therefore $\sim M\{(x, \infty)\}$, as asserted.

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