Since T_j is of the form $T_j = a_j I + K_j$, it follows from (25) that (37) is equivalent to

$$[P(\lambda_1) I - \sum_{j=1}^{m} \lambda_1^{m-j} K_j] u_0 = 0 . \tag{38}$$

Since $P(\lambda_1) \neq 0$ by hypothesis, this is a generalized Fredholm equation of the second kind. The number λ_1 is an eigenvalue of $\mathbf{T}$ if and only if (38) has a non-zero solution u_0, in which case (36) gives a corresponding eigenvector $\vec{u}_0$.

A special case of a composite recursion relation was studied by D. Greenspan and the author in [4]. Asymptotic results were obtained there which go beyond those given above.

We conclude this paper with a generalization of Theorem 1. Let $\mathfrak{A}$ be an algebra with unit I over the complex field. Let $\mathscr{I}$ be an ideal in $\mathfrak{A}$. Let $\mathfrak{A}_m$ denote the algebra of all $m \times m$ matrices $\mathbf{T} = [T_{ij}]$ with $T_{ij} \in \mathfrak{A}$. Then the set

$$\mathscr{I}_m = \{\mathbf{K} = [K_{ij}] : K_{ij} \in \mathscr{I}\} \tag{39}$$

is an ideal in $\mathfrak{A}_m$.

Theorem 2. Let $\mathbf{T} = [a_{ij} I + K_{ij}]$, where $K_{ij} \in \mathscr{I}$. Let $P(\lambda)$ be the characteristic polynomial of the scalar matrix $[a_{ij}]$. Then $P(\mathbf{T}) \in \mathscr{I}_m$.

Since the proof is essentially the same as that for Theorem 1, it is omitted.

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