7. Desirability of a new orientation of mathematical teaching.

Objekttyp: **Chapter**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **1 (1955)**

Heft 1-2-3: **L'ENSEIGNEMENT MATHÉMATIQUE**

PDF erstellt am: **18.09.2024**

(" rules ") than to prove them. This reminds of the English school-boy having to learn " Euclid ", and saying to his teacher " Bother the proofs. Tell me the results ! "

Another prevailing opinion holds that the secondary education should restrict itself to " pure " mathematics, without bothering much about the applications. This, however, disregards the fact, mentioned already before, that most of " pure " mathematics is old " applied " mathematics. In particular this holds for school mathematics, with the exception of geometry in its Euclidean form: logarithms were introduced purely as a computational method; the solution of algebraic equations by means of roots is due to the fact that roots (from positive numbers) were originally almost the only onevalued functions one could master (this same fact gave rise later to Galois' theory; from a modern computational point of view it is without the slightest importance whether an equation can be reduced to successive extractions of roots, whereas formerly this was highly relevant); trigonometry was introduced as an expedient for astronomical, nautical and geodesical problems; descriptive geometry was introduced by Monge as a method to avoid the at that time very clumsy computational methods. Descriptive geometry is, by the way, the only subject on the mathematics curriculum which is less than about 300 years old; it is even less than two centuries of age !

In this context the fact should be mentioned that the Educational Institute at the University of Utrecht has successfully initiated an experiment in teaching elementary probability theory and history of mathematics in the highest classes of literary gymnasia, and to abolish the teaching of solid geometry and of broken linear functions in these schools. The experiment will be continued with other school-types also.

7. Desirability of a new orientation of mathematical teaching.

The large extent of obligatory mathematical education for *all* pupils in most schools is usually justified, apart from its applications, by 1° stating that " instruction in mathematics furthers logical thinking ", 2° implying that this is a desirable aim for all pupils, and 3° accepting a hypothesis called the " principle of transfer ", which may be expressed by stating that the faculty of logical thinking, if exercised on special subjects like geometry and algebra only, is automatically " transferred " to applications to other subjects also.

For the sake of argument we shall take here the first statement for granted, by disregarding the vagueness of the term and the objections which could be made against it, but which would lead too far away from the main subject. The second statement undervalues the important difference between deductive and inductive logic, and misjudges the fact that purely deductive logic is only applicable within the context of a mathematical model, and that by purely deductive reasoning no non-trivial empirical statement about observable phenomena can ever be proved or disproved. As to the " principle of transfer ", much has been written about it, but the present author is not aware of serious efforts to test this hypothesis empirically in a way satisfying modern standards of research.

Such an investigation should in any case go into the following remarks, which are based on the personal experience of the author only, and therefore, of course, can not be considered as conclusive.

The attitude of mathematicians towards problems which are rather far from the ordinary mathematical sphere, and which can not be tackled by means of deductive logic, seems not to be very different from that taken by other intellectuals, except that the tendency to avoid them may be somewhat stronger among mathematicians. On the one hand it seems that the mathematician's attitude towards them on the whole is rather intelligent and often based on broad human feeling, and that the more extremistic and in particular the more irrational attitudes are not frequent among them. On the other hand a considerable degree of aloofness from political and philosophical questions can be observed among mathematicians, which might point to a feeling of helplessness towards problems where " logic of partial knowledge " is involved and where data are lacking for making treatment on a rigorous base possible. Among those, however, who do not avoid these questions, one finds, notwithstanding the positive qualities mentioned above, only very rarely that the main features of their mathematical work are maintained in this work also. In particular the main characteristic of mathematicians, viz. to take the utmost precautions against wishful thinking and other forms of self-deception can hardly be said to find its counterpart in the context of other activities of the same mathematicians. Thereby it becomes possible that so many political and religious creeds, each accepting a body of statements, which, if pooled, contains numerous contradictions, so that they certainly can not be true all, nevertheless have among their adherents mathematicians, even of the highest quality, or other scientists which have had an intensive mathematical training. Apparently whatever form of automatic transfer may exist, it is insufficient to break through the emotional and traditional background of such creeds, unless the individual is willing and has been trained (or trained himself) in reasoning as " logically " as possible in cases also where insufficient data together with strong emotions are present. Another instance, pointing in the same direction is the fact that most mathematicians, when discussing the value of mathematics, do not, or hardly, consider the possibility that this need not be always positive, or at least do not try to find all serious arguments which might be brought forward for the alternative possibility.

Moreover, although I might not underrate the importance of systematic study of mathematics, and of one of its main characteristics, viz. to separate difficulties and mastering these one after another, one wonders that mathematicians seem not to be able to transfer this characteristic to their educational problems. For, otherwise, how could one understand that mathematical courses do not contain separate parts and groups of exercises for training and testing *separately* the different objectives one has in mind, like acquiring mathematical techniques, theoretical insight, systematizing ability, inventiveness and ability of correct logical reasoning, but that all these elements are mixed up within almost *every* exercise ?

Resuming this argument, we might state that it is at least very doubtful whether training in mathematics, based on deductive logic, leads automatically to an increased capacity of arguing logically in cases where only inductive reasoning is possible, and where often only quite insufficient data, together with strong emotions and/or traditions are present. It seems rather that a special training in the latter direction is necessary. This, however, would make it necessary to revise the " epistemological " basis upon which obligatory mathematical training for *all* students, apart from their respective needs for applications, could be justified. On the other hand this, of course, does not exclude the possibility—which the present author considers as very probable—that deductive and inductive reasoning are sufficiently close in order that teaching of mathematics, provided it will be adapted to the revised needs, may be very useful for the purpose.

Regarding the form of re-orientation of mathematical teaching necessitated by the preceding arguments we might make the following remarks.

1. In the first place is needed: a precise and differentiated formulation of objectives of instruction, using operationally defined terms instead of rather vague terms like " furthering logical thinking ", etc., so that it is possible to test with respect to every pupil, *whether* and/or *in how far* the objective has been reached in his case. The differentiation of objectives should at least entail that *a)* ability to apply special mathematical techniques; *b)* correct ideas about particular theoretical considerations; *c)* systematizing data as well as purposes of an investigation, and following an appropriately chosen orderly line of thought; *d)* inventiveness in overcoming new difficulties, and *e)* correct logical reasoning, either according to the rules of deductive logic (proofs of mathematical statements), or to the less strict rules of inductive or " plausible " inference, can be taught *separately* and tested *separately.*

2. The differentiation of purpose should correspond with a differentiation according to the individual capacities, individual interest, and the professional future of the pupils. Evidently pupils going later to a household-school or getting a job in a

post office or police-HQ, those who go to the university to study law or languages, who go to an engineering school, who are going to study medicine, biology, pharmacology, economy, psychology or social sciences, and those who will become astronomers, physicists, or mathematicians, have quite different needs.

3. This differentiation should be reflected in a differentiation of requirements for the final high school examination, which at present, in the Netherlands, are identical for very large groups of students.

As a final summing up, I believe I may say that we as mathematicians should take care that the mass product we produce, viz. the results of our students, admit a satisfactory quality control, that the results we pretend we can obtain can be subjected to the requirements of testability which the statistician demands from every research worker in biology or medicine, that we are aware of the restricted reliability of our tests (examinations) and admit definite tolerance limits, but also that we know how to balance the "yield", differentiated according to different requirements, against the "cost" in the form of teaching- and learning-hours, and know to treat this as a decision problem.

This seems to me to be a duty of honour for us as mathematicians.

APPENDIX

Examples of modern problems in different fields where mathematics is applied [1]

A. *Statistical applications in medicine, biology and pharmacology.*

1. An epidemiological investigation of tuberculosis in Indonesia.
2. Biological standardization of insulin by experiments on rabbits.

[1] The examples are taken from problems treated in the Mathematical Centre at Amsterdam.