Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 61 (2015)
Heft: 3-4

Rubrik: ICMI Study 23 : primary mathematics study on whole numbers

L'Enseignement Mathématique (2) 61 (2015), 505–517

COMMISSION INTERNATIONALE DE L'ENSEIGNEMENT MATHÉMATIQUE

(THE INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION)

ICMI Study 23: *Primary Mathematics Study on Whole Numbers*

Maria G. Bartolini Bussi* and Xuhua Sun**

Abstract. In this paper[1], we report on the progress of the first ICMI Study directing attention on primary level mathematics, focused on whole number arithmetic. We describe the background and the planning of the study, the realization of the study conference and the preparation of the volume emanating from discussions at the conference, due for publication at the end of 2016.

1. Background

Primary schooling is compulsory in all countries, although with different facilities and opportunities for children to take advantage of it. One in three children never see the inside of a classroom. In developing countries considerable proportions of children in third grade do not master expected levels of reading comprehension. Mathematics is a central subject in primary mathematics education (together with language) and the delivery of the mathematics curriculum is important in all countries for the kinds of citizens and the kinds of competences each seeks to produce. The ubiquity of mathematics in primary school education is of interest in relation to the dominance of secondary school mathematics education in previous ICMI studies. Hence a reflection on primary school mathematics was considered timely by the ICMI Executive Committee (term 2010–2012). An outline of the discussion in the Executive Committee (EC) is reported below:

In the international literature there is much research and writing on primary school mathematics. In many cases, especially in the West, early processes of mathematical thinking are

* Università di Modena e Reggio Emilia (Italia)
** University of Macau SAR (China)

[1] This paper draws on the draft version of the forthcoming volume edited by Bartolini Bussi and Sun (in preparation).

investigated by cognitive psychologists, who study the emergence of these processes in laboratory settings, where children are stimulated to observe, for example, the emergence of one-to-one correspondence, counting, or measuring. This is interesting but not sufficient to cope with the more complex situation of schools, with institutional and cultural constraints. In several countries (in Europe and North America), the influence of Piaget, although criticized in research literature, is still very deep and strong. In other parts of the world different models are adopted.

The study of this key core content area is often regarded as foundational for later mathematics learning. However, the principles and main goals of instruction in the foundational concepts and skills are far from universally agreed upon, and practice varies substantially from country to country. An ICMI Study that provides a meta-level analysis and synthesis of what is known about primary mathematics will provide a base from which to gauge silences and an opportunity to learn from different countries and contexts.

A reflection on primary mathematics education will not be complete without considering the context and conditions in which mathematics teaching and learning takes place. In developing countries, primary schooling for a vast majority of children may be continually interrupted, and takes place in overcrowded, under-resourced large classrooms of substandard quality. A part of the study may be devoted to the analysis of some case studies. Primary mathematics curriculum policy reforms in different countries and the effectiveness of their implementation in classroom practice would also address a diversity of audiences.

2. The scope of the study

In recognition of the above arguments, the theme of the study was defined as follows:

The beginning of the approach to whole numbers, including operations and relations, and the solution of arithmetic word problems, in schools (and possibly pre-school environments), up to grade 3 or more, according to the various education systems.

Whole number arithmetic (WNA) was chosen by the EC of ICMI, based on their consideration of the necessity of selecting a topic focus with a shared centrality in primary school mathematics curricula all over the world, although it is not the only topic relevant to primary school mathematics.

The study was lauched by ICMI at the end of 2012, with the appointment of two co-chairs and of the International Program Committee (IPC), which, on behalf of ICMI was responsible for conducting the study:

Maria G. (Mariolina) Bartolini Bussi – Italy and Xuhua Sun — Macau SAR China (co-chairs)

Berinderjeet Kaur – Singapore
Hamsa Venkat — South Africa
Jarmila Novotná – Czech Republic
Joanne Mulligan – Australia
Lieven Verschaffel – Belgium
Maitree Inprasitha – Thailand
Sybilla Beckmann – US
Sarah González – Dominican Republic
Abraham Arcavi – Israel (ICMI Secretary-General)
Ferdinando Arzarello – Italy (ICMI President)
Roger E. Howe – US (ICMI liason)

During 2013 an intense mail exchange realised within the IPC established and shared the rationale, the goals and the steps of the forthcoming study. In January 2014 an IPC meeting took place in Berlin, at the IMU Secretariat, which generously supported the costs. The IPC members were welcomed by Prof. Dr. Jurgen Sprekels, director of the Weierstrass Institute for Applied Analysis and Stochastic (WIAS, Berlin), and by the ICMI President Prof. Ferdinando Arzarello, who participated in the whole meeting, and, later, in the whole Study Conference.

The meeting in Berlin took place in a productive and collaborative climate. Five themes (each corresponding to a Working Group in the Conference) were identified and assigned to pairs of

members of the IPC (later, Christine Chambris agreed to substitute for Sarah González, who was unable to take part in the Conference due to health reasons):

(1) *The why and what of whole number arithmetic*

(2) *Whole number thinking, learning, and development*

(3) *Aspects that affect whole number learning*

(4) *How to teach and assess whole number arithmetic*

(5) *Whole numbers and connections with other parts of mathematics*

Three plenary panels were also identified:

(1) *Traditions in whole number arithmetic*, chaired by Ferdinando Arzarello

(2) *Special needs in research and instruction in whole number arithmetic*, chaired by Lieven Verschaffel

(3) *Whole numbers arithmetic and teacher education*, chaired by Jarmila Novotná

Three plenary speakers were invited:

(1) Hyman Bass: *Whole numbers: Quantities, numbers, number names, and the real number line*

(2) Brian Butterworth: *Low numeracy: from brain to education*

(3) Ma Liping: *Whole Number Arithmetic in primary school: Different perspectives on the "same" subject, their intellectual roots and possible impacts*

The intention of the IPC was to offer a map of some important issues relating to WNA, crossing the borders of countries and regions. The aim was to foster reflection among participants (and, subsequently, among the readers of the volume) on their own cultural contexts, with representation in the conference and the volume of sources from a wide range of geographical and socioeconomic contexts. Cole's 1998 book on Cultural Psychology affirms the need for this kind of range:

> *In recent decades many scholars whose work I discuss have sought to make the case for a culture-inclusive psychology. They argue that so long as one does not evaluate the possible cultural variability of the psychological processes one studies, it is impossible to know whether such processes are universal or specific to particular cultural circumstances. For examples, John and Beatrice Whiting, anthropologists with a long-term interest in human development, wrote: 'If children are studied within the confines of a single culture, many events are taken as natural, or a part of human nature and are therefore not considered as variables. It is only when it is discovered that other people do not follow these practices that have been attributed to human nature that they are adopted as legitimate variables'* (p. 2).

The temptation of a narrow and local perspective is a risk for mathematics educators too, given the enormous advantages that mathematics developed in the West has had on the development of science, engineering and technologies. This study aimed at challenging some of these beliefs with a short, yet lively, immersion in an atmosphere where a more open mind is needed at least when discussing early years mathematics, and where the strong links with everyday life and cultural traditions come into play.

A discussion document[2] including a call for papers for the Study Conference was prepared, with the Study Conference announced for June 2015 in Macau (SAR China). This document summarised issues that were considered important to discuss in the study. As noted already,

[2] http://www.umac.mo/fed/ICMI23/dd.html

special emphasis was given to the importance of cultural diversity and to the effects of this diversity on the early introduction of whole numbers. In order to foster understanding of the different contexts in which authors had developed their studies, each applicant for the study was required to include background information about their submission's context.[3]

3. The Study Conference and the Study Volume

By the end of the selection process, 67 papers were accepted and distributed over the five themes. For each accepted paper a maximum of two co-authors were invited to participate in the Study Conference. A volume of proceedings was edited by Xuhua Sun, Berinderjeet Kaur and Jarmila Novotná (Sun, Kaur, and Novotná, 2015).

Thanks to generous support from the University of Macau SAR, the Education and Youth Affairs Bureau, Macau SAR and from ICMI, for the first time the ICMI 23 study was able to invite observers from non-affluent countries. A choice was made to privilege *Capacity & Networking Project* (*CANP*) *and The Mathematical Sciences and Education in the Developing World* participants who are the major developmental focus of the international bodies of mathematicians and mathematics educators. Other observers came from the Great Mekong area and from China. The total number of participants was 91 from twenty-two countries.

The Study Conference was held in June (3-7) 2015 in Hengqin Campus, University of Macau leased to Macau by the State Council of the People's Republic of China, in 2009, for the construction of the new campus. The conference was opened by Prof. Zhao Wei, Rector of the University of Macau with addresses by Mr. Wong Kin Mou, Representative of Director of the Education and Youth Affairs Bureau, Chief of Department of Research and Educational Resources of Macao SAR, by Prof. Shigefumi Mori, president of IMU, Prof. Ferdinando Arzarello, president of ICMI, and the co-chairs (the co-authors of this paper). A beautiful gallery of photos from the Study Conference is available on the conference website.[4]

The ICMI Study Conference served as the basis for the production of the Study Volume, that is in preparation and will be presented on the occasion of ICME 13[5] in Hamburg (2016). Discussions drawing from the Working Groups, Plenary Panels and the Plenary presentations during the conference have found their place in the Study Volume. A short summary of the chapters of the volume follows.

4. The Study Volume: The Introductory Chapters

4.1. Social and cultural contexts in the teaching and learning of whole number arithmetic. This chapter stands witness to the growing importance of the need to know the social and cultural context in the teaching and learning of primary mathematics, outlining the background of the studies carried on in the past decades. The aim of this chapter is to report on the process that led the IPC to require the preparation of background information (the so-called *context form*) for each submitted paper, for the first time in an ICMI Study and, also, in the major international conferences. A short analysis of all the completed context forms is then presented and commented upon. The data represent a convenience sampling, rather than the population of mathematics educators as a whole. While the information provided is in the format of contributors' perceptions of their broader national contexts, the data provides useful background detail on broad

[3] http://www.mathunion.org/fileadmin/ICMI/docs/CONTEXT_form.pdf
[4] http://www.umac.mo/fed/ICMI23/photo.html
[5] http://icme13.org

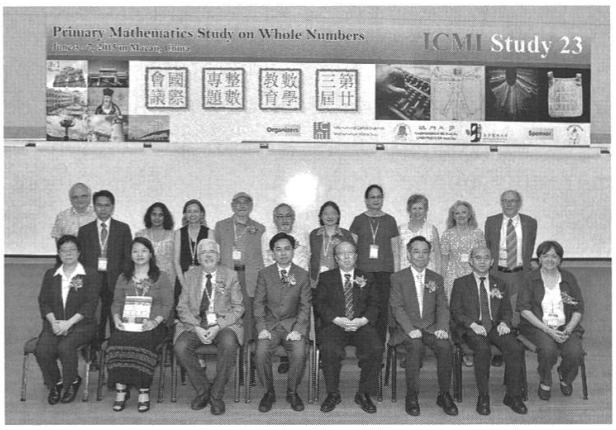

FIGURE 1
The official photo at the opening session at the University of Macau (SAR China). From right to left in line 1 : Maria G. BARTOLINI BUSSI, Xitao FAN, Shigefumi MORI (President Of IMU), Wei ZHAO (The Rector), Kin Mou WONG, Ferdinando ARZARELLO (President of ICMI), Xuhua SUN, I Lin WONG.
From right to left in line 2 : Lieven VERSCHAFFEL, Joanne MULLIGAN, Jarmila NOVOTNÁ, Berinderjeet KAUR, Liping MA, Brian BUTTERWORTH, Hyman BASS, Sybilla BECKMANN, Hamsa VENKATAKRISHNAN, Maitree INPRASITHA, Abraham ARCAVI (ICMI Secretary General).

differences between countries in terms of the structure and organization of primary schooling, and of the nature of mathematics curricula and teacher preparation within systems.

4.2. Language issues in the teaching and learning of whole number arithmetic. This chapter addresses issues of language that emerged in some working groups. Two aspects are highlighted: the influence of local languages on the words of WNA and the ways of using technical terminology (e.g. "carrying" or "borrowing") that have become accepted in some communities even if they do not seem to be the most appropriate terms for the construction of mathematical meanings. This chapter is also cross-linked to the former ICMI Study 21 on *Mathematics Education and Language Diversity* (Barwell et al., 2016), but its limited focus on WNA allows us to give more details about the different languages and cultures associated with number of the participants in the Conference. WNA is universal in the sense that every culture includes it. What is not universal is the approach or understanding of it. While the assumption of universality of approaches has been predominant for both curriculum reformers and international evaluators for many years, this chapter discusses examples to show that WNA is not culture-free, and deeply rooted in local languages and culture with the inherent difficulty of transposition from one language to another, and from one culture to another.

5. The Study Volume: The Working Group Chapters

Five chapters are devoted to the outcomes of the five working group discussions. Their themes are frequently connected to each other and cross-links are highlighted in the volume.

5.1. The why and what of whole number arithmetic (WNA). The chapter begins by providing a historical perspective that captures some of the roots of WNA and its development in different contexts around the globe. In different cultures humans identify needs and develop material and intellectual tools in order to address them. The tensions between needs and means engaged to address them results in the invention of more and more sophisticated numerical practices. Various examples are given and key comparisons are made, especially in relation to the epistemological genesis of numbers. The second section highlights the interactions between cultural identities including language and universal decimal features of WNA. Furthermore colonial issues are explored where the gaps between spoken and written numbers are juxtaposed with numeration and calculation. The third section highlights the processes within societies throughout history when attempts at changes are made in WNA. Through societal changes, several stakeholders assign different goals for WNA that imply tensions between communities.

5.2. Whole number thinking, learning, and development. This chapter focuses on the neurocognitive and cognitive and developmental analyses of WNA learning. It comprises four sections. The first section reviews neurocognitive perspectives on learning WNA and draws on the mental number line but looks beyond to explain the transcoding of numerals to number words. In the second section, children's early mathematics-related competence in reasoning about quantitative relations, patterns and structures are explored from new theoretical perspectives. Studies presented and discussed in the working group are introduced as exemplars of intervention studies from cognitive perspectives in the following section. The last section examines methodologies utilized in studies on children's whole number learning and development. It discusses study designs and their potentialities and limitations for understanding how children develop competencies with whole numbers, and task designs in cognitive neuroscience research pertinent to number learning.

5.3. Aspects that affect whole number learning. In this chapter aspects that affect whole number learning are considered from two complementary perspectives: aspects that may help learning, especially if adequately exploited by the teacher; and aspects that may hinder learning, especially if not adequately contrasted by the teacher. Students' cultures may help or hinder the development of mathematical thinking. This difference depends also on the fact that what is considered mathematics at the international level is the result of a process that has been developed mainly in Europe, according to the societal needs and language constraints of European society during past centuries. The participants discussed several issues (language, artifacts, tasks) but agreed that the core of the discussion in the working group was the notion of artifact: this notion and the related notion of signs, models, teaching aids, and manipulatives are developed in the chapter focusing on traditional cultural artifacts and on concrete and virtual artifacts including the recent development of multitouch technologies. Some examples of widespread epistemological obstacles are discussed, together with artifacts, which may reduce their impact on the teaching and learning process. A section on mathematical tasks is included, with links to ICMI Study 22 on *Task Design* (Watson and Ohtani, 2015).

5.4. How to teach and assess whole number arithmetic. This chapter focuses on the diverse theoretical and methodological frameworks that capture the complex relationship between whole number learning, teaching and assessment. Its aim is to bring these diverse perspectives into conversation. The importance of the theme for developing pupils' understanding of mathematics, and their overall learning, is obvious. Across the world there are multiple approaches to WNA teaching and assessment, with differences seen within various countries as well. While it is not feasible to describe several approaches in one chapter of a book, the authors instead present one concrete example of school practice – a WNA lesson observed by ICMI Study 23 participants

at a primary school in Macau, China – and use the lesson as a stimulus for discussing ideas about teaching, learning and assessing WNA. The focus for all sections is how teachers promote the development of pupils' metacognitive strategies during their learning of WNA. Each section develops an important aspect of the theme.

5.5. Whole numbers and connections with other parts of mathematics. This chapter concerns presentation/discussion of structure and structuring activities as two key routes through which WNA can be connected to other mathematical content areas and to central mathematical processes and products like defining/definitions and generalizing/generalization. In the body of the chapter, literature is used to distinguish between approaches focused more on the presentation of structure and those oriented towards structuring activities, before presenting an overview and discussion of studies geared more towards one or other of these approaches. Studies have been incorporated that have been directed towards both students' mathematical learning and mathematical (and pedagogical) teacher learning. A commentary on biases towards structure-based or structuring activity-based approaches across these contexts is appended. The argument is that both approaches show promise for building towards stronger connections between WNA and other mathematical areas, with a number of examples in each category included. Given the evidence of difficulties for so many children in many parts of the world in moving beyond the terrain of whole number, the findings suggest that attention to structure and structuring can provide important routes for bridging this chasm.

6. The Study Volume: The Panel Chapters

The three panels aimed to address some transversal issues that cut across the Working Group foci, with the participation of most members of the IPC exploiting their areas of expertise, and of some other invited participants. The names and the short contributions of the panelists are available in the Study Conference Proceedings (Sun, Kaur, and Novotná, 2015, pp. 603–624).

6.1. Traditions in whole number arithmetic. Ferdinando Arzarello, President of ICMI, chaired a panel on traditions in WNA. Panelists reported on verbal and non-verbal representations of numbers and artefacts for arithmetic. Some issues about the pragmatics of whole numbers were reported showing the many different uses of numbers in everyday life and possible inconsistencies between everyday meanings and mathematical meanings. The issue of numbers in colonial countries where the school language is often different from traditional local languages was discussed. The contrast between some popular cultural artifacts in the West (the number line) and in the East (the suànpán, see Fig. 2, added in 2013 to UNESCO's list of intangible heritage) was discussed, relating to differences in cultural tradition. Historic-epistemological analysis shows that the uses of either artifact are strongly related to the deep values of Western and Chinese mathematics, with different emphasis in these traditions on continuous vs. discrete quantities (see also the plenary speeches by Hyman Bass and Ma Liping below). Hence the number line and the suànpán are cultural artifacts that reveal valuable information about the society that made or used them and, when continuity between tradition and today's practices is maintained, foster the students' cultural awareness of the role mathematics played in their society, that is their cultural identity. The discussant was Siu Man Keung from Hong Kong University, who discussed the different traditions developed in West and East and their meeting at the beginning of the 17th century when Matteo Ricci, from Italy, travelled to Macau and later to mainland China.

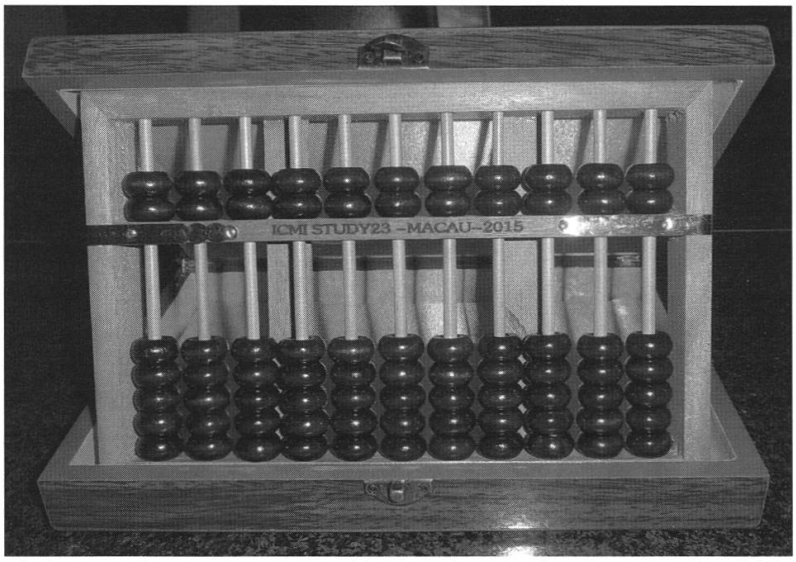

Figure 2
The customized suànpán for the participants in the Conference

6.2. Special needs in research and instruction in whole number arithmetic. Lieven Verschaffel, member of the IPC, chaired a panel on special needs in the domain of WNA. In the panel the need for agreement in the definition of mathematical learning difficulties (MLD) was addressed, highlighting the lack of coherence and consensus about what constitutes "mathematics" in MLD. Within MLD research there is a history of predominance of focus on memorization of arithmetic facts and automatization of arithmetic procedures. A less (neuro)psychologically dominated and more interdisciplinary approach might bring a broader, more coherent and balanced perspective that takes into account both the views about mathematics learning as arithmetic and other equally important perspectives such as spatial and geometrical reasoning, mathematical relations and patterns, and other forms of mathematical thinking with more potential towards abstraction and generalization. Some studies were reported concerning both the diagnosis of MLD and interventions in MLD. In particular, some panelists addressed the question of whether students with MLD should be educated in the inclusive classroom or segregated instructional environment, and whether these children need a special kind of intervention or profit more from the same kind of instruction as children without MLD. The discussant was Brian Butterworth, from London University College, a well-known scholar of developmental dyscalculia.

6.3. Whole number arithmetic and teacher education. Jarmila Novotná, a member of the IPC, chaired a panel on teacher education. The main goal of the panel, as well as of the chapter, was to explore approaches to, and within, primary mathematics teacher education in different parts of the world and to discuss commonalities and differences in relation to broader cultural and curricular traditions. There was broad agreement that deep understanding of school mathematics in general and WNA in particular (for primary teachers) is critical. It follows from this that WNA provides a critical context for developing understandings and constructing arguments that adhere to the practices and norms of more advanced mathematics. A key thrust of this chapter was to present and discuss examples from several parts of the world that approach the need to attend to mathematical norms and practices through a focus in primary mathematics teacher education on developing, discussing and applying mathematical models. But this thrust was set within the broader terms of different curricular approaches to WNA in different countries and regions, and different cultures and structures regulating the ways in which primary mathematics

teacher education (and primary teacher education more generally) are organized. The discussant was Mike Askew, an established and widely cited scholar in primary school mathematics and primary mathematics teacher development with experience in England, Australia, South Africa and Chile.

7. The Study Volume: The Plenary Speeches Chapters

The three plenary speeches were aimed at addressing WNA from three different perspectives: a mathematician's perspective; a neurologist's perspective; and a mathematics educator perspective with deep knowledge of two different research traditions.

7.1. Whole numbers: Quantities, numbers, number names, and the real number line. Hyman Bass, past president of ICMI and a research mathematician, addressed an approach to developing concepts of number using general notions of quantity and their measurement. This approach, most prominently articulated by Davydov and his colleagues, is discussed, and some arguments favoring this approach are offered. The first argument is that it provides a coherent development of both whole numbers and fractions. Second, it makes the geometric number line continuum present from the start of the school curriculum as a useful mathematical object and concept into which real numbers can eventually take up residence. Third, in the Davydov approach there are significant opportunities for some early algebraic thinking. In addition, a further instructional context and approach to place value that simulates a hypothetical invention of a place value system for number representation is reported.

7.2. Low numeracy: from brain to education. Brian Butterworth introduced a neuropsychological perspective on the theme. It is widely agreed that humans inherit a numerical competence, though the exact nature of this competence is disputed. The author argues that it is the inherited competence with whole numbers (the 'number module') that is foundational for arithmetical development. This is argued on the basis of a longitudinal study of learners from kindergarten to Year 5. Recent research has identified a brain network that underlies our capacity for number and arithmetic, with whole number processing forming a core region of this network. A twin study shows a strong heritable component in whole number competence, its link to arithmetical development and to the brain region. These findings have implications for improving numeracy skills, especially among low-attaining learners.

7.3. Whole number arithmetic in primary school: Different perspectives on the "same" subject, their intellectual roots and possible impacts. Ma Liping, with the coauthor Kathy Kessel, addressed the different perspectives on WNA in the US and China. In the US, the tendency is to consider WNA as consisting of learning to compute the four basic operations with whole numbers. In contrast in China, WNA involves much more than simply learning to carry out the computational algorithms. For example, it is expected that students explore the quantitative relationships among the operations, and represent these (sometimes quite sophisticated) relationships with (sometimes quite complicated) numerical equations. This exploration of quantitative relationships is made possible by the theoretical core that underlies school arithmetic. In this article, she presented the central pieces of this theoretical core.

8. Concluding remarks

8.1. Merits of the Study. The Study Volume is an account of the collective memory of participants offered to the wide community of primary mathematics educators, including researchers, teachers, teacher educators and policy makers. It is a product of fruitful cooperation between mathematicians and mathematics educators, when, for the first time in the history of ICMI, the issue of whole numbers arithmetic in primary school has been addressed. Many authors, beside the ones mentioned in this article, are involved in the volume. The panelists and some of the participants in the working groups contributed and are acknowledged as coauthors of the chapters. Moreover, expert scholars of the field kindly agreed to write commentary papers for some of the chapters: Bernard Hodgson, Roger Howe, Claire Margolinas, John Mason, Pearla Nesher and David Pimm. While they did not take part in the Conference, their broad expertise in the mathematics education field was sought with a view to connecting the specifics of the papers located in WNA in primary mathematics with this broader field, and thus offering a different perspective on our key themes.

In all the working groups and the panels, the discussion was lively and the presence of the Chinese culture was evident: the colleagues from the Chinese areas discussed their own perspectives, often different from the others' and still connected with the classical tradition. The ICMI 23 conference was located in the ideal place: Macau is known as the place of a dialogue between Portugal and China, between European and Eastern cultures. This dialogue is evident not only in the architecture of the old city, but also of the new Hengqin campus; in the roadsigns (written in Chinese and Portuguese); in the cultural institutions such as the Ricci Institute, that the participants visited as a part of the social program, with lectures about the role of Matteo Ricci in introducing elements of European mathematics into China (Bartolini Bussi and Sun, in press).

Figure 3
The photo of the participants (together with Confucius) after the closing session.

A relevant and interesting part of the social program was the visit to two first grade classrooms, to observe lessons on addition and subtraction, according to the typical Chinese tradition of *open classes*, where many observers (many dozens in our case) assist with a lesson, with a carefully organized teaching plan distributed in advance, and discussed with the teachers later in order to improve the lesson for the future. The participants showed great interest in this lively observation of a Chinese classroom that is described at length in one chapter of the Study Volume. The immersion in a culture so different from that of most participants led to a sharing of some

features of a range of different tradition, providing a much broader and deeper airing of some of the issues that had been outlined in the brevity of the textual summary format of the context form described above.

The attention to contexts and to different cultural traditions is one of the major merits of this study, and in place from the beginning in the Discussion Document. Our sense is that it could (and should) attract the interest of policy makers. In this trend, it is worth noting our entrenching of what previously has been incorporated more as a 'special interest' rather than a core feature: the plenary panel on *Cultural contexts for European research and design practices in mathematics education* (Jaworski et al., 2015) hosted by the CERME 9 (the Conference of the European Society for Research on Mathematics Education, held in Prague in 2015) and the plenary address by Bill Barton on *Mathematics education in its cultural context: Plus and minus thirty years* to be given in ICME 13. The directions seem to us to be right but the way is still long.

A further innovation related to our central attention to culture was that during the conference, in some working groups, short video clips about classroom episodes were shown by the participants, who had agreed to prepare them with English subtitles. The vivid impression that a video clip can give of classroom life and of the implicit culture is different from what is discernible in a written paper. The IPC had hoped to collect many video clips, but it is likely that some authors did not have the permissions needed to meet all the privacy and ethical rules, and/or the time to prepare the English trancripts required to make the video clips understandable in the context of an international conference. We are planning, however, to collect a gallery of video clips that might be enlarged in the future. References to video-clips appear in the volume in appropriate places.

8.2. Limits of the Study. In spite of the huge efforts of the IPC members, a limitation of the study was the failure to involve mathematics educators from a wider range of countries and regions (e. g. Russia, Japan, Korea, most parts of Africa and Latin America). China was well represented. This is not surprising because of their proximity to the Study Conference venue. However, equity imperatives in the participation in this ICMI study remain far from being reached, although the themes of the conference had the potential to involve mathematics educators from non-affluent countries and policy makers. Several obstacles may be identified: ineffective dissemination: international mailing lists and journals continue to reach a limited portion of the mathematics education community across the world; language issues: the choice of English as the study language, although unescapable, might have inhibited some authors to apply; costs: airfares are not likely to be strictly related to the distance from countries, but are rather controlled by commercial constraints.

In spite of this caveat, the aim of constructing a map of the main educational aspects of WNA has been partially fulfilled in the conference and in the volume, with a wide multicultural approach. Some themes have been deepened further, and some others have been opened up as new avenues that currently are simply sketched. For instance, the issue of textbooks within the teaching of WNA has been touched upon in one of the chapters, but would deserve a whole study in its own. Another theme that deserves further exploration is assessment of, and for, WNA learning. The ICMI study on assessment is very old (Niss, 1992 a,b) and the situation is changed internationally (e.g., the OECD PISA) with important influences also at the country level. The issue of gifted student was only skimmed when challenging mathematical tasks were considered. Hence, in this case too, there is space for further development.

8.3. Impact of the Study. The impact of the study is promising. Some communities have shown their interest already before the Study Conference, inviting members of the IPC to report on the study process (e.g. Beckmann, 2014). After the conference, reports (by invitation) have appeared in some journals (European Mathematica Society Newsletter, Mathematics Education

Journal, in Chinese, Bulletin of liason CFEM in French) and conference proceedings (Copirelem, Bartolini Bussi and Sun, in press; SEMT 2015, Novotná, 2015).

The CANP representatives have acknowledged the importance of the study conference, where each of them was assigned to a working group, ensuring that a dialogue between the other participants and the observer was developed. They also had a formal meeting with the ICMI President, Prof. Ferdinando Arzarello, during which, for the first time, experiences across CANPs were shared. Veronica Sarungi (personal communication), representative of CANP 4, noted in her reflections:

> *'One of the major contributions of the 23rd ICMI study was to enable the CANPs to build networks beyond their regions. As a result of connections formed in Macau, a discussion group proposal was submitted and accepted for ICME-13 that will focus on CANPs. Apart from networking, the meeting in Macau enhanced the individual capacity of the representatives that had an effect on their respective institutions, national and regional associations.'*

The intercultural dialogue between mathematics educators interested in WNA for the primary school, will continue in International Conferences (such as SEMT, taking place every second year in Prague[6]) and in ICME. Moreover, the Interamerican Conference on Mathematics Education (IACME), taking place every four years, has a special section on Primary Mathematics Education and WNA is an important part of it.

The presentation of the Study Volume will be held in a special timeslot at ICME 13. We do hope that the special focus on WNA in primary school mathematics within ICMI Study 23 lays the ground for further attention to primary mathematics topics, curricula and pedagogies in the future to be addressed in the conferences of the affiliated organization to ICMI, because *building the foundation*, as the title of volume reads, is critically important for the development of mathematics teaching and learning in secondary schools and beyond.

Acknowledgements. The preparation of this report was shared with the whole extended IPC, as was every step of the study process.

References

Bartolini Bussi, M. G. and X. Sun, eds. (in preparation), *Building the foundation: whole numbers in the primary grades.* New York-Berlin: Springer.

Bartolini Bussi, M. G. and X. Sun (in press), "Learning from the world: the teaching and learning of whole numbers arithmetic in the icmi study 23." In *Proceedings of the 42nd colloque Copirelem*, Besançon.

Barwell, R., P. Clarkson, A. Halai, M. Kazima, J. Moschkovich, N. Planas, M. Setati, P. Valero, and M. Villavicencio, eds. (2016), *Mathematics education and language diversity. The 21st ICMI study.* New ICMI Studies Series, New York-Berlin: Springer.

Beckmann, S. (2014), "The twenty-third icmi study: primary mathematics study on whole numbers." *International Journal of STEM Education*, 1, 1–8, URL `http://dx.doi.org/10.1186/2196-7822-1-5`.

Cole, M. (1998), *Cultural psychology: A once and future discipline*. Harvard University Press.

[6] `http://www.semt.cz`

Jaworski, B., M. G. Bartolini Bussi, S. Prediger, and E. Nowinska (2015), "Cultural contexts for european research and design practices in mathematics education." In *Proceedings of the CERME 9-Ninth congress of the European society for research in mathematics education* (K. Kreiner and N. Vondrova, eds.), 7–35, Prague: Charles University in Prague, Faculty of Education and ERME.

Niss, M. (1992 a), *Cases of assessment in mathematics education: An ICMI study*, volume 1. New York-Berlin: Springer.

Niss, M. (1992 b), *Investigations into assessment in mathematics education: an ICMI study*, volume 2. New York-Berlin: Springer.

Novotná, J. (2015), "Presentation of the ICMI Study 23." URL `http://www.semt.cz`. Retrieved February 10, 2016.

Sun, X., B. Kaur, and J. Novotná, eds. (2015), *Conference proceedings of the ICMI study 23: primary mathematics study on whole numbers*, URL `http://www.umac.mo/fed/ICMI23/proceedings.html`. Retrieved February 10, 2016.

Watson, A. and M. Ohtani, eds. (2015), *Task design in mathematics education. The 22nd ICMI Study*. New York: Springer.