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ICMI study *Task design in mathematics education*: An outline of progress

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1. Background

Over the last few decades there has been an increasing focus on disseminating, researching and critiquing task design in mathematics education. Schoenfeld (1980) and Sierpinska (2003) plead for more communication between designers and researchers. Schoenfeld (1980) makes the point that many designers are not articulate about their design principles, and may not be informed by research. Sierpinska (2003) points out that research reports rarely give sufficient detail about tasks for them to be used by someone else in the same way. Few studies justify task choice or identify what features of a task are essential and what features are irrelevant to the study. In some studies using intervention/treatment comparisons to investigate cognitive development, the intervention tasks are often vague, as if the reader can infer what the learning environment was like from a few brief indications. As an example of how a task can be invisible to researchers, we could look at the commentaries about a well-known and widely accessible video of a Japanese mathematics lesson for the TIMSS study (`http://www.timssvideo.com/67`). The problem involves a crooked boundary between two areas, and the task is to straighten it. The teacher mentions several times in post-lesson discussion that by tilting their heads the students can see the problem as similar to work they have done previously on areas of triangles between parallel lines. In the commentaries reported on the website, it is only the teacher who mentions connections between the task design, its presentation, and students' participation; the researchers appear to talk only about the social and structural features of the lesson. Yet the task features are central to the success of the lesson in terms of lesson structure and learning, because the teacher goes on to present a similar 'tilted' viewpoint, using the same underlying geometric principle, in a further task. Furthermore, this task

[0] This paper draws on the Discussion Document for ICMI Study 22 (`http://www.mathunion.org/icmi/digital-library/icmi-discussion-documents/`), and Chapter 1 of the forthcoming book Watson and Ohtani (in press).

has been included in every official textbook for at least 40 years, and the diagram that goes with it is one the students are already familiar with, albeit from a different viewpoint. These features of the task are, we believe, crucial to understanding the lesson and the students' mathematical responses, but are hardly mentioned.

In 2008, the International Congress on Mathematics Education (ICME11) hosted a topic study group (TSG), *Research and development in task design and analysis*, which provided a forum for the necessary interaction between design and research. Designers were asked to be explicit about their principles and demonstrate how they used them, and proceedings are still available at http://tsg.icme11.org/tsg/show/35. The TSG increased its membership during the conference, indicating that a serious, organised look at task design was of growing interest, and subsequent ICMEs have continued this TSG.

Other sites of relevant scholarly activity have also been increasing. Burkhardt and Schoenfeld have founded the International Society for Design and Development in Education and a journal, *Educational Designer* (www.educationaldesigner.org). There are several well-established working groups and conferences on digital task design and digital learning environments (e.g., *International Conference on Technology in Mathematics Teaching*; Clark-Wilson et al., 2013), and there is also recent growth of similar work on other aspects of design. For example, there are publications about:

- the effects of task design on learning and assessment (e.g., Runesson, 2005);
- publication of tasks, principles of design, and research on effects and implementation by longstanding design teams (e.g., Shell Centre, UK; Freudenthal Institute, Netherlands; QUASAR, US; Connected Mathematics, US);
- changes in task design at implementation stage (e.g., PME research forum: Tzur et al., 2008) ;
- the process of didactic engineering and the influence of tasks on teaching (e.g., Margolinas et al., 2011);
- international textbook comparisons that draw attention to differences in task design (e.g., Valverde et al., 2002);
- task design and use in teacher education (e.g., *Journal of Mathematics Teacher Education*, volume 10 (4-6); Tirosh and Wood, 2009; Zaslavsky and Sullivan, 2011);
- how curriculum is enacted through the use of textbooks (e.g., Thompson and Usiskin, 2014).

2. The scope of the Study

In recognition of this growth of interest, a proposal was constructed for ICMI Study 22: *Task design in mathematics education* and presented to the International Commission on Mathematics Instruction (ICMI) in 2011, and was accepted. Co-convenors and an International Programme Committee (IPC) were appointed from across the world and met in Oxford early in 2012. The IPC felt that the Study needed to embrace not only the processes of task design by well-known teams, but also other aspects.

It should include recent interest in the role of tasks in teacher education (Sullivan, 1999). Mathematics teacher education, as a subfield of mathematics education, has paid significant recent attention to the nature, role and use of tasks with a triple special issue of the *Journal of Mathematics Teacher Education* (volume 10, 4-6) edited by Mason, Watson and Zaslavsky, and a book edited by Zaslavsky and Sullivan (2011). A volume of the *Handbook of Mathematics Teacher Education* has been devoted to the tasks and processes of teacher education (Tirosh and Wood, 2009).

The Study should aim to recognise the interface between design and practice. A research forum at PME in Mexico (Tzur et al., 2008) offered cogent arguments for the desirability and inevitability of teachers' adaptation of the cognitive demand of tasks. Further, Choppin (2011) suggests how adaptation differs among teachers. In variation theory (Marton et al., 2004) a helpful distinction is made between the intended, enacted, and lived objects of learning. The Documentational Approach of Didactics (Gueudet and Trouche, 2009, 2011) also refers to the practitioner perspective in terms of the resources on which teachers draw. Didactic engineering was the topic of the 15th summer school in mathematics didactics in 2009 (Margolinas et al., 2011). The discussion focused not only on various principles of task design but also on the problem of the influence of task design on the development of actual mathematics teaching.

The study should aim to be international. While the Anglophone world may be familiar with the success of theoretically-based long term design-research projects resulting in publications such as those from Shell Centre (Swan, 1985), Realistic Mathematics Education (de Lange, 1996), Connected Mathematics (Lappan and Phillips, 2009) and the QUASAR project (Quantitative Understanding: Amplifying Student Achievement and Reasoning) (Henningsen and Stein, 1997), it may not be so familiar with the Francophone approach to didactics as outlined above, nor the detail of how task design and pedagogy interact in typical Pacific Rim classrooms.

The study should also recognise the norms of mathematics education internationally. Most mathematics learners world-wide learn procedures and possibly concepts through 'practice', regardless of the de-emphasis on procedures held by reform enthusiasts. Thus, the design of sequences of near-similar tasks deserves attention. For reasons of global reality and equity, the study conference should also focus on textbook design partly because textbooks are often informed by tradition or by an examination syllabus rather than through research and development (Valverde et al., 2002), but also because in some countries textbooks are the major force for change. Textbooks are not the only medium in which sequences of tasks, designed to afford progressive understanding or shifts to other levels of perception, can be presented, and we expected that study conference participants would look also at the design of online task banks.

The IPC organised these considerations under five themes, each theme having shared responsibility among two or three of the IPC members. Introductory text and orienting questions for each theme were prepared by the IPC members:

2.1. Theme A: Tools and representations. There has always been a close relationship between mathematical tasks and the tools available to support that work. This theme focused on the interactions between tool use, task design and students' learning, and how designers think about these connections. It also addressed how new technologies can provide new ways for teachers and learners to engage with core mathematical ideas.

2.2. Theme B: Accounting for student perspectives in task design. The actual impact of task design on students' mathematical learning raises important questions. One of the aims of this thematic group was to gain insights into students' perspectives and to better understand how appropriate task design might help to minimise any differences between teacher intentions and student learning.

2.3. Theme C: Design and use of text-based resources. The design and use of tasks presented in textbooks and other published resources is central to many school students' experience of mathematics. This theme was to address the design, content and sequencing of such resources and the way they shape the mathematics curriculum and what it means to learn and do mathematics.

2.4. Theme D: Principles and frameworks for task design within and across design communities. Papers for this group were to address the major theories of learning, mathematical activity and design, displaying their principles and ways in which these are enacted. Theme E: Features of task design informing teachers' decisions about goals and pedagogies The intention of this group was to synthesise what is known about teachers' decision making about task use, and hence to offer suggestions about task design for teachers, teacher educators, task designers, text and resource authors, and curriculum developers. A discussion document and call for papers were prepared for a study conference announced for July 2013 in Oxford[1]. The document summarised issues that might be discussed in the study, as above, and also framed some common ground: the meaning of 'task'; the scope of design of tasks and sequences of tasks; design communities and methods.

3. The meaning of 'task'

The word 'task' is used in different ways. Some writers (Christiansen and Walther, 1986) express 'task' as being what students are asked to do. Then 'activity' means the subsequent mathematical (and other) motives that emerge from interaction between student, teacher, resources, environment, and so on around the task. 'Task' sometimes denotes designed materials or environments which are intended to promote complex mathematical activity (e.g., Becker and Shimada, 1997). We decided to use 'task' to mean a wider range of 'things to do' than this, and include repetitive exercises, constructing objects, exemplifying definitions, solving single-stage and multi-stage problems, deciding between two possibilities, or carrying out an experiment or investigation. Indeed, a task is anything that a teacher uses to demonstrate mathematics, to pursue interactively with students, or to ask students to do something. Task can also be anything that students decide to do for themselves in a particular situation.

4. Task design

The extent and detail of design varies widely. For some (e.g., Shell Centre) design includes full necessary materials, task sequences and advice about effective choices, and detailed pedagogic advice about ways of working, verbal interventions, likely misconceptions and possibly extensions. For others (Ainley et al., 2005), there may be provision of a question, or a microworld, or some physical material, with no written object to describe 'the complete task', but rather a series of things that the teacher might say, perhaps supported by some written prompts. Another form of design is to refine a problem-situation until it is most likely to promote intriguing mathematical reactions. All of these approaches have implications for implementation, with some relying on teachers' existing skills, some providing advice to extend teachers' skills, and others dependent on teachers maintaining or adapting the designer's intentions (see, e.g., Kieran et al., 2011).

It is important to address also the question of sequences of tasks. There are different aspects embedded in the design of sequences and, while this is an obvious consideration when designing textbooks, it also stretches across the whole field of task design. To achieve the goal of teaching a whole conceptual field we have to describe the different aspects of this knowledge and the way the aspects are linked (for interesting examples about rational number see Brousseau et al., 2004a,b, 2007, 2008, 2009). The earlier tasks in a sequence should provide experiences that

[1] http://www.mathunion.org/fileadmin/ICMI/files/Conferences/ICMI_studies/Ongoing_studies/ICMI_Study_22_announcement_and_call_for_papers_and_discussion_document.pdf

scaffold the student in the solution of later tasks, allowing them to engage in more sophisticated mathematics than would otherwise have been the case. Stated like this it seems self evident, yet many multi-authored textbook series do not make these links, presenting instead a fragmented sequence of topics, images and ideas.

There are different ways to create sequences of tasks, some of them are more commonly known by teachers themselves (e.g., Huang and Bao, 2006; Yoshida, 1999). One type of task sequences is that in which the problem formulation remains constant but the numbers used increase the complexity of the task, say moving from small positive integers (for which answers might be easy to guess) to other ranges of numbers for which a general method might be needed. Another type of sequence is one in which the problem is progressively made more complex by the addition of steps or variables, such as in a network task where additional nodes are added. A third type of sequence may be one where the concept itself becomes more complex, such as in a sequence of finding areas or progressively more complex shapes from rectangles, to composite shapes, to irregular shapes.

The importance of sequencing is explicit in Realistic Mathematics Education. In that tradition, a task sequence starts with situated problems (Gravemeijer, 1999) and continues by changing the focus to formalizing and generalizing solution procedures. In this type of task sequence guidance from informal to formal concepts is important (Van den Heuvel-Panhuizen, 2003).

5. Design communities and methods

Design can involve designers, professional mathematicians, teacher educators, teachers, researchers, learners, authors, publishers and manufacturers, or combinations of these, and individuals acting in several of these roles. In the diverse communities and methods that lead to the development and use of tasks all methods, the central consideration is the interaction between teachers and learners through the designed artefacts and/or the design process. A major focus in the study will therefore be on learning how design impacts on learners and learning, rather than research, which focuses solely on the design process.

6. Questions

The theme groups presented between 5 and 10 questions each to orientate discussion at the conference. However, the point of the study was not only to answer predetermined questions, and not merely to synthesise existing research, but to go beyond and set an agenda for future work. In particular, we asked contributors to be explicit about the ways in which teachers, designers and researchers worked together in their contexts, so that the study could think about issues beyond those raised in published research. There was a robust response and the study conference was limited strictly to 80 attenders in order to ensure effective working groups and effective interaction between groups. Papers were selected to ensure an optimal coverage of relevant issues[2]. One author from each paper was invited to attend the conference. In addition, four plenary speakers were invited, each representing a well-established, significant approach to the principles, practice, and implementation of task design from across the world. While work on task design is considerably more advanced in some countries than in others, and the conference was conducted in English, nevertheless it was possible to construct an international delegacy presenting work from North and South America, Europe, Nordic countries, Pacific Rim and Australia, with special presentations

[2] https://hal.archives-ouvertes.fr/hal-00834054

from Japan, Netherlands, Spain and Israel. The medium term intention of the study was not the conference itself, but the production of a state of the art book about task design in mathematics education, based in discussions and further scholarly work on the five themes and the special presentations. The book has now been completed and is in press (Watson and Ohtani, in press). It represents much more than a summary of papers presented at the conference or answers to pre-determined questions; it provides a critical synthesis of the relationships between principles and practices of design, the associated pedagogies, and the classroom interactions that frame students' learning. The original themes have been adapted and reorganised to make a coherent book, rather than a collection of separate papers. We now summarise the contents:

6.1. Frameworks and principles for task design. Frameworks and principles for task design address three theoretical *grain sizes*, although a specific set of principles might incorporate different sizes. The grain sizes identified in the Study are: *grand frames*; *intermediate frames*; *domain specific frames*. Grand frames present theories about learning in and out of educational settings at a general level. Intermediate frames present the complex interactions between task, teacher, teaching methods, educational environment, mathematical knowledge, and learning so that the purposes and implications for task design are always understood within the total structure of practice. Communities can develop around both grand and intermediate theories in which there is shared language, shared materials and resources, shared research studies and conferences. An extra dimension of intermediate frames is that they are based in teaching as craft knowledge and arise from teachers' actions and interactions. A particularly useful contribution made by this theme group is a summary of the history of task design in mathematics education. Very often in our field, people only refer to recent research and recent experiences of practice. Also, because much of our practice is influenced by policy, ideology, and (at the time of writing) the international testing regime, it is tempting to refer to those as the basis for scholarly work rather than our own past research.

6.2. Teachers' decisions arising from design and pedagogy. Sometimes people refer to "gaps" between what is intended by the designer and enacted by the teacher, or what is intended by the teacher and perceived by the learner. Such gaps can also be seen more positively as "interactions" which are inevitable in the teaching/learning process. The related theme group posed practical questions about the influence of task features on teachers' decision-making and had available to them a wide range of reports about how teachers use and adapt tasks. They discussed suitability and applicability, and how these are influenced by teachers' views of the nature of mathematics, the prevailing school and classroom culture, and broader goals a teacher may be pursuing with the students. The teacher's knowledge of mathematics pedagogy and ability to anticipate students' responses is critical in all these decisions. The authors had originally intended to address related issues about educating teachers in the processes of using tasks, but these varied so widely between cultures and teacher education programs that they could not do it justice within the confines of this study.

6.3. Accounting for student perspectives in task design. Merely doing what the teacher hopes and expects is evidence for a certain form of student compliance, but might not constitute evidence of learning, or evidence of understanding the purpose of the task or even evidence of having the same perception of the task as that of the teacher. The question the authors of this chapter wanted to address is how learners answer this question: "what is this task asking me to do?" The group became aware of a dearth of research in this area, and yet knowledge of how learners perceive a task is crucial to planning effective lessons as well as to designing effective tasks. *Perception* therefore has to be imagined, and in some cultures the expertise of the teacher is seen in terms of the accuracy of that process of imagination. The theme group therefore concentrated first on the literature about word problems, which is extensive and draws attention to differences in students' perceptions of the purpose of the task and designers' intentions. The group used the concepts of *didactical situation* as a structure within which to consider the *didactical contract*, that is the tacit agreement between teacher and student about what was required. They proposed ways in which various educators, teachers, and researchers have sought to reduce any 'gaps' between the teacher's intentions and the learner's perceptions, such as reflective re-design; considerations of openness of tasks; and the idea of *emergent task design*. Emergent task design is a process arising from ideas of learners, in the course of a lesson, which are made into tasks by teachers spontaneously.

6.4. Design issues related to text-based tasks. This theme group set out originally to focus on the design of tasks in textual format, textbooks more generally, downloadable materials, and other forms of text-based communication designed to generate mathematical learning. The group was aware of differences in the order, development, representation, and presentation of content between textbook series, and also between countries and cultures. Few papers were submitted that addressed these questions. Instead, the theme received many papers about designed tasks or collections of tasks, generally addressing overarching aspects of students' mathematical learning, such as proof, interdisciplinary perspectives, reasoning, problem solving, values. A few were more specific. Meanwhile, the *International Conference on Mathematics Textbook Research and Development 2014* was announced, signalling an increase in international comparison, cooperation, and knowledge exchange about mathematics textbooks, their design, development, use and analysis. The expectations for this new conference freed the ICMI Study 22 working group to focus on issues raised by the conference papers, and others that could not be addressed at the level of textbook production and use.

The group conceptualised a mutually interactive relationship between the nature and structure of the task, the intended mathematical activity, and the pedagogic purpose. It referred throughout to tasks that are free-standing as well as those within learning *management systems*, meaning published textbooks, task banks, programmed systems and so on. The triangular relationship is relevant for thinking about free-standing tasks, home-made task banks, and textbooks, whether digitally-delivered or paper-based, and tasks created during lessons.

6.5. The role of tools. This theme concerns designing teaching/learning tasks that involve the use of tools in the mathematics classroom and consequently how, under such design, tool use represents mathematical knowledge. The issue for designers is how to relate tool-specific representations to mathematical knowledge, and this is a subfield of mathematics education that is well-established. There have been several journals, special issues and conferences since the 1980s. For our study, the submitted papers were mainly concerned with practical and theoretical issues of task design in dynamic digital environments, but also included papers on nondigital tools.

The working group introduced the idea of the discrepancy potential of a tool, which is the space between the feedback a learner might experience from using the tool and the intended mathematics concept. They also constructed a synthesis of the issues that any tool-based task design heuristics need to address: complementarity of feedback and mediation; relationships between pragmatic and epistemic considerations; symbiosis of mathematics and pedagogy; multiplicity of tools; and the discrepancy potential just described. The design of the task needs to coordinate these aspects of the activity.

In all working groups the importance of the teacher's perception of the nature of mathematics was emphasised.

7. Plenaries

The invited plenaries added exemplification and depth to the considerations above[3], and each presentation has given rise to a book chapter. Michal Yerushalmy from Israel describes the theory and design of her digital resource that focuses on various features of functions for secondary students. One interesting feature of this resource, and the reason she was chosen to give a plenary, is that she embraces the facility of digital technology to provide flexible rather than pre-determined sequencing of tasks. Berta Barquero and Marianna Bosch illustrate how they have used the Theory of Didactic Situations, which is derived from the work of Guy Brousseau in France, at primary level to establish the measurement of quantities, and then they demonstrate the more complex world of didactic engineering in a development of the original theory. Toshiakira Fujii describes an aspect of Japanese Lesson Study, *kyozaikenkyu*, which is the process by which teachers spend considerable time researching and designing tasks and teaching, and reading existing research and learning from each other and from experts. This approach, where task design is the central focus for teachers' training, planning and development, runs counter to beliefs of some specialists who state that design should be left in the hands of designers. Jan de Lange, an experienced designer whose work in the Netherlands at the Freudenthal Institute has been influential throughout the world, gives a down-to-earth and practical description of specialist task design, setting high standards for the use of intuition and insight, informed by mathematical knowledge, as starting points and practical experiences in the classroom with teachers and learners as essential aspects of the process. For him, the nature and direction of research about task design is in danger of moving away from direct experience of children and mathematics in classrooms and towards theorization.

8. Looking ahead

Our main concern in leading this study has been to accelerate the growth of attention to task design given by researchers in their work and their written artefacts. We have achieved that. The main outcome is the forthcoming book from Springer (Watson and Ohtani, in press). Several papers that contributed to the conference have now been expanded and developed and published elsewhere, and have included these where possible in the relevant reference lists. A volume about task design with digital technologies follows the study (Leung and Baccaglini-Frank, forthcoming) as does a special issue of the Journal of Mathematics Teacher Education edited by Keith Jones and Birgit Pepin. A research forum took place in 2014 on the relationships between task and students (Clarke et al., 2014).

For design teams, the work of designing, trialling, and publishing has, in the past, rightly taken priority over reporting the design research processes in an internationally accessible way, or researching their own practice, and the degree to which they expected teachers to understand their background theoretical justifications varies. Teachers all over the world might be familiar with the task of graphing the heights achieved by filling bottles of various shapes, or the task of estimating the size of the giant given the dimensions of the handprint, or the task of enlarging the drawing of a rectilinear animal. Teachers use these tasks, not because they are committed to the precise background theory that led to their invention, nor because their use has been researched and theorized in some other classrooms and countries. Rather, teachers use these tasks because they match the practices involved in local coordination of curriculum demands, classroom practices, intended mathematical outcomes, and anticipated participation of particular individuals and groups of students.

[3] http://www.mathunion.org/icmi/digital-library/icmi-study-conferences/icmi-study-22-conference/

There is a need for detailed research reports that are not unhelpfully limited in length and can fully report studies of design of tasks as well as the associated pedagogy which is usually influenced by local values more strongly than background theory. Details of tasks and the likely effects of task design features, as well as pedagogy, should be included more frequently in research about classrooms and learning. We point also to a need for researchers to distinguish between theories *of* their observations, and theories *for* designers and teachers, and to consider drawing on teachers' and learners' perspectives when theorizing in either case. As for theories of task design, evaluations of effectiveness are always going to take place in natural contexts consisting of specific classrooms, teachers, constraints and cultures, so it is inevitably the case that empirical studies will not be extensively generalizable, but can be illuminative and give rise to conjectures.

As editors, we pose some areas for further research that arose from the study, sometimes from several theme groups:

- how learners/teachers make sense of, and understand the purpose of, different kinds of tasks;
- how different design principles reflect or generate different perceptions of mathematical concepts;
- how different combinations of tasks and pedagogy influence learners' perceptions and mathematical activity;
- how visual features of task presentation affect activity;
- the design, implementation and effects of task sequences;
- the professional learning of teachers about task design, sequencing and adaptation;
- the role of task design in promoting equity and other values;
- task design and individual learner differences;
- the effectiveness of forms of collaboration and communication between task designers, classroom teachers, professional mathematicians, educators, and policy-makers.

Finally, Ken Ruthven and Michèle Artigue provide critical commentaries offering further insights and suggestions for future research.

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