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Denote by $\mathcal{E}(0)$ the sheaf of analytic pseudo-differential operators in $T^*\mathbf{C}^n \approx \mathbf{C}^{2n} = \{(q,p)\}$ of order 0. Let $\mathcal{E}'(0)$ be a subsheaf of operators which depend only on some of the variables, say $q_1, \dots, q_j, p_1, \dots, p_k$. We denote by $\mathcal{E}_0(0)$, $\mathcal{E}'_0(0)$ the stalks of the sheaves $\mathcal{E}(0)$, $\mathcal{E}'(0)$ at the point $x_0 \in \mathbf{C}^{2n}$ with coordinates $q_1 = \cdots = q_n = 0$, $p_1 = 1$, $p_2 = 0, \dots, p_n = 0$.

THEOREM 6.8 ([3, 33]). *For any coherent left $\mathcal{E}_0(0)$-module M the following assertions are equivalent:*

1. *the $\mathcal{O}_{\mathbf{C}^{j+k-1},0}$-module $M/\partial_{q_1}^{-1}M$ is of finite type;*
2. *the $\mathcal{E}'_0(0)$-left module M is of finite type.*

Proof. The module M is the stalk at the point $x_0 = (0, \dots, 0, 1, 0, \dots, 0)$ of a sheaf $\mathcal{M}$ of $\mathcal{E}_0(0)$-modules in $T^*\mathbf{C}^n \approx \mathbf{C}^{2n}$.

Consider the complex given by a resolution of $\mathcal{M}$

$$\mathcal{K}^\bullet : \ \cdots \xrightarrow{\delta_2} \mathcal{E}(0)^{n_1} \xrightarrow{\delta_1} \mathcal{E}(0)^{n_0} \longrightarrow 0\,, \quad \mathcal{H}^\bullet(\mathcal{K}^\bullet) = \mathcal{E}(0)^{n_0}/\mathrm{Im}\,\delta_1 \approx \mathcal{M}\,.$$

The support of $\mathcal{M}$ coincides with that of $\mathcal{M}/\partial_{q_1}^{-1}\mathcal{M}$, it is therefore an analytic subvariety $V \subset \mathbf{C}^{2n}$ ([35]; see also [33], Proposition 4.2.0).

The restriction of the sheaf $\mathcal{E}(0)$ to the complement of the zero section in $T^*\mathbf{C}^n \approx \mathbf{C}^{2n}$ is a sheaf of non-commutative Fréchet algebras [4]. Therefore the argument given in the proof of Proposition 6.7 applies *mutatis mutandis* to this situation. □

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