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One could now use some results on Kleinian groups to conclude that $\alpha_n = 1$ (see, for example [7, I.D.4 and II.C.6]) but we prefer to give the following elementary argument. A simple computation shows that

$$\gamma_n^{-1} \begin{bmatrix} 1 & 1/m \\ 0 & 1 \end{bmatrix} \gamma_n = \begin{bmatrix} 1 & \frac{1}{m\alpha_n} \\ 0 & 1 \end{bmatrix}, \quad \gamma_n \begin{bmatrix} 1 & 1/m \\ 0 & 1 \end{bmatrix} \gamma_n^{-1} = \begin{bmatrix} 1 & \frac{\alpha_n}{m} \\ 0 & 1 \end{bmatrix}.$$

These two transformations are in $\Gamma \cap \mathcal{T}$, and so α_n and $1/\alpha_n$ must be (positive) integers ; that is, $\alpha_n = 1$. But then γ_n will be in $\Gamma \cap \mathcal{T}$ and therefore $z_n = z'_n$, since $X = \mathbf{H}/(\Gamma \cap \mathcal{T})$.

This proposition gives us a second way to prove the Big Picard Theorem without using the modular function, as follows. As in the proof given above, the function $f: \mathbf{D}^* \rightarrow X = \widehat{\mathbf{C}} \setminus \{\infty, 0, 1\}$ lifts to a function $\tilde{f}: \mathbf{D}^* \rightarrow \mathbf{D}^*$ that gives a commutative diagram :

$$\begin{array}{ccc} & \mathbf{D}^* & \\ \nearrow \tilde{f} & & \downarrow \rho \\ \mathbf{D}^* & \xrightarrow{f} & X. \end{array}$$

The function \tilde{f} has a removable singularity at the origin with $\tilde{f}(0) = 0$; the problem is to determine the behaviour of the map ρ . By Proposition 4.5 there is an ϵ such that ρ is a finite covering from \mathbf{D}_ϵ^* onto its image. But then by the Casorati-Weierstrass theorem ρ cannot have an essential singularity at the origin, and neither can f .

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