Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 55 (2009)
Heft: 1-2

Artikel: The Big Picard theorem and other results on Riemann surfaces
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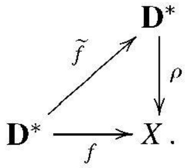
Bibliographie
DOI: https://doi.org/10.5169/seals-110098

One could now use some results on Kleinian groups to conclude that $\alpha_n = 1$ (see, for example [7, I.D.4 and II.C.6]) but we prefer to give the following elementary argument. A simple computation shows that

$$\gamma_n^{-1} \begin{bmatrix} 1 & 1/m \\ 0 & 1 \end{bmatrix} \gamma_n = \begin{bmatrix} 1 & \frac{1}{m\alpha_n} \\ 0 & 1 \end{bmatrix}, \qquad \gamma_n \begin{bmatrix} 1 & 1/m \\ 0 & 1 \end{bmatrix} \gamma_n^{-1} = \begin{bmatrix} 1 & \frac{\alpha_n}{m} \\ 0 & 1 \end{bmatrix}.$$

These two transformations are in $\Gamma \cap \mathcal{T}$, and so α_n and $1/\alpha_n$ must be (positive) integers; that is, $\alpha_n = 1$. But then γ_n will be in $\Gamma \cap \mathcal{T}$ and therefore $z_n = z_n'$, since $X = \mathbf{H}/(\Gamma \cap \mathcal{T})$.

This proposition gives us a second way to prove the Big Picard Theorem without using the modular function, as follows. As in the proof given above, the function $f \colon \mathbf{D}^* \to X = \widehat{\mathbf{C}} \backslash \{\infty, 0, 1\}$ lifts to a function $\widetilde{f} \colon \mathbf{D}^* \to \mathbf{D}^*$ that gives a commutative diagram:



The function $\widetilde{f}$ has a removable singularity at the origin with $\widetilde{f}(0) = 0$; the problem is to determine the behaviour of the map ρ. By Proposition 4.5 there is an ϵ such that ρ is a finite covering from $\mathbf{D}^*_\epsilon$ onto its image. But then by the Casorati-Weierstrass theorem ρ cannot have an essential singularity at the origin, and neither can f.

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(Reçu le 28 juin 2007; version révisée reçue le 8 janvier 2008)

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