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Other scenarios for uncertain information include those in which the pursuers do not admit an initial all-to-all communication round, but rather communicate with pursuers which are sufficiently close. Far-off pursuers cannot be reached. This and similar problems touch on many ideas currently in play in the control theory literature on distributed consensus with limited/faulty communication [35].

REMARK 28 (*Other noncooperative pursuit games*). There are numerous examples of pursuit-evasion games beyond the *Lion & Man* setting: see [12, 25] for an overview. We mention in particular the case considered by Isaacs [12] in which the evader's goal is to reach a specified subset of the domain. More recent entries in the literature consider pursuit games in which capture means not physical coincidence, but rather visibility — the pursuer wins when there is a line-of-sight to the evader. For results in this genre, see [34, 9]. More recently, much attention has been paid to probabilistic techniques in pursuit games: see [13, 14, 37].

Stepping back from the game-theoretic perspective, one can consider a pursuit-evasion game as a form of cooperative consensus problem, where a "swarm" of pursuers attempts to reach positional consensus with an evasive "leader". Consensus problems have received a great deal of attention recently from the control-theory community, with motivation from biologically observed swarming phenomena. Several authors [5, 35, 23] have given decentralized algorithms for reaching consensus in a variety of contexts.

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