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$\mathbf{C}S_n$ is the centralizer of $GL_k(\mathbf{C})$ on $W^{\otimes n}$, for $k \geq n$, where $W = \mathbf{C}^k$ is the representation of $GL_k(\mathbf{C})$ by matrix multiplication on column vectors. If we replace a simple tensor with the subset indexed by the binary string in its subscripts — for example $v_1 \otimes v_0 \otimes v_1 \otimes v_1 \otimes v_0 \Leftrightarrow 10110 \Leftrightarrow \{1,3,4\}$ — then the action on simple tensors is the same as the action of P_n on subsets in Section 2.

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