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TABLE 1
Normal surface in the figure eight knot complement

solution	$\nu(\mu)$	$\nu(\lambda)$	slope
$(2,0,0,0,0,1)$	1	4	−4
$(0,2,0,0,0,1)$	−1	4	4
$(0,0,1,2,0,0)$	−1	−4	−4
$(0,0,1,0,2,0)$	1	−4	4

quadrilateral types in M' by r, r', r'', where $r^{(i)}$ lifts to $p^{(i)}$. The Q-matching equation is $r + r' - 2r'' = 0$. It can be worked out from the triangulation or by observing that the induced involution on quadrilateral types in M is $(p\ q')(p'\ q)(p''\ q'')$. Thus, $\dim PQ(\mathcal{T}') = 1$ and $PF(\mathcal{T}') = \varnothing$.

The boundary curve map is defined via the induced triangulation of the double cover of the Klein bottle; using the generators from the above section, one has: $\nu(\lambda) = -2r - 2r' + 4r'' = 0$ and $\nu(\mu) = -2r' + 2r''$. Generators λ', μ' can be chosen for $H_1(B_\mathfrak{v}) \cong \mathbf{Z} \oplus \mathbf{Z}_2$ such that the map $H_1(\widetilde{B}_\mathfrak{v}) \to H_1(B_\mathfrak{v})$ is given by $\lambda \to \lambda'$ and $\mu \to (\mu')^2 = 0$. The composition

$$Q(\mathcal{T}') \to \mathbf{Z}^2 \to H_1(\widetilde{B}_\mathfrak{v}; \mathbf{R}) \to H_1(B_\mathfrak{v}; \mathbf{R})$$

is then

$$N \to (-\nu_N(\lambda), \nu_N(\mu)) = (0, \nu_N(\mu)) \to \nu_N(\mu)\lambda \to \nu_N(\mu)\lambda' .$$

Since $\nu(\mu) = -2r' + 2r''$, it follows that the map $\partial \colon Q(\mathcal{T}') \to H_1(B_\mathfrak{v}; \mathbf{R})$ is surjective. Its restriction to integral points in $Q(\mathcal{T}')$ has image of index two in $H_1(B_\mathfrak{v}; \mathbf{Z})$, which gives a subgroup of index four in $H_1(B_\mathfrak{v})$.

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