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TABLE 1  
Normal surface in the figure eight knot complement

solution	$\nu(\mu)$	$\nu(\lambda)$	slope
(2, 0, 0, 0, 0, 1)	1	4	-4
(0, 2, 0, 0, 0, 1)	-1	4	4
(0, 0, 1, 2, 0, 0)	-1	-4	-4
(0, 0, 1, 0, 2, 0)	1	-4	4

quadrilateral types in  $M'$  by  $r, r', r''$ , where  $r^{(i)}$  lifts to  $p^{(i)}$ . The  $Q$ -matching equation is  $r + r' - 2r'' = 0$ . It can be worked out from the triangulation or by observing that the induced involution on quadrilateral types in  $M$  is  $(p\ q')(p'\ q)(p''\ q'')$ . Thus,  $\dim PQ(\mathcal{T}') = 1$  and  $PF(\mathcal{T}') = \emptyset$ .

The boundary curve map is defined via the induced triangulation of the double cover of the Klein bottle; using the generators from the above section, one has:  $\nu(\lambda) = -2r - 2r' + 4r'' = 0$  and  $\nu(\mu) = -2r' + 2r''$ . Generators  $\lambda', \mu'$  can be chosen for  $H_1(B_v) \cong \mathbf{Z} \oplus \mathbf{Z}_2$  such that the map  $H_1(\tilde{B}_v) \rightarrow H_1(B_v)$  is given by  $\lambda \rightarrow \lambda'$  and  $\mu \rightarrow (\mu')^2 = 0$ . The composition

$$Q(\mathcal{T}') \rightarrow \mathbf{Z}^2 \rightarrow H_1(\tilde{B}_v; \mathbf{R}) \rightarrow H_1(B_v; \mathbf{R})$$

is then

$$N \rightarrow (-\nu_N(\lambda), \nu_N(\mu)) = (0, \nu_N(\mu)) \rightarrow \nu_N(\mu)\lambda \rightarrow \nu_N(\mu)\lambda'$$

Since  $\nu(\mu) = -2r' + 2r''$ , it follows that the map  $\partial: Q(\mathcal{T}') \rightarrow H_1(B_v; \mathbf{R})$  is surjective. Its restriction to integral points in  $Q(\mathcal{T}')$  has image of index two in  $H_1(B_v; \mathbf{Z})$ , which gives a subgroup of index four in  $H_1(B_v)$ .

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