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Other examples should be tested to confirm, or invalidate, d_3 as a rightful analogue of prime natural density in \mathbf{Z} .

We conclude by a general question pertaining to the density theory in number systems presented in the book [Bu]. Consider a class \mathcal{K} of finite structures and a property \mathcal{P} . Define, for each $n \geq 1$, \mathcal{K}_n as the subset of these structures of size n . Define p_n as the proportion of structures in \mathcal{K}_n that have property \mathcal{P} , and P_n as the proportion of structures in $\bigcup_{k=1}^n \mathcal{K}_k$ that have property \mathcal{P} . In [Bu], the function p_n (resp. P_n) is referred to as the *local* (resp. *global*) *counting function* and is somewhat akin to the d_1 (resp. d_2) approximant of order n . General results [Bu] have been proved in which the existence of a limit for p_n , or P_n , as $n \rightarrow \infty$ has been established. Suppose we define an *average counting function* \bar{p}_n as $\frac{1}{n} \sum_{k=1}^n p_k$. Are there classes of structures such that neither p_n , nor P_n have a limit law, but \bar{p}_n does? Can we build a general theory of limit laws based on the function \bar{p}_n ? This function being similar to the d_3 -approximant of order n , Propositions 1.11 and 1.13 of our paper suggest that if \bar{p}_n converges there are general hypotheses under which the associated Dirichlet density also exists.

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