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# 65

## Finiteness Properties of Function-Field-Arithmetic Groups

by Kevin WORTMAN

Let  $K$  be a global function field, and  $S$  a finite nonempty set of pairwise inequivalent valuations on  $K$ . We let  $\mathcal{O}_S \leq K$  be the corresponding ring of  $S$ -integers, and we let  $\mathfrak{G}$  be a simple  $K$ -group. For any  $v \in S$ , we let  $K_v$  be the completion of  $K$  with respect to  $v$ .

The finiteness properties of function-field-arithmetic groups such as  $\mathfrak{G}(\mathcal{O}_S)$  have been of interest for about 50 years. Some of that interest has fallen on which of these groups are of type  $\text{FP}_m$  for a given  $m$ . For example, for which  $m$  is it true that  $\text{SL}_n(\mathbf{F}_q[t])$  is of type  $\text{FP}_m$  where  $\mathbf{F}_q[t]$  denotes a polynomial ring with one variable and coefficients in a finite field with  $q$  elements.

We recall that a group  $\Gamma$  is *of type  $\text{FP}_m$*  if there exists a partial resolution

$$P_m \rightarrow P_{m-1} \rightarrow \cdots \rightarrow P_1 \rightarrow P_0 \rightarrow \mathbf{Z} \rightarrow 0$$

of the trivial  $\mathbf{Z}\Gamma$ -module  $\mathbf{Z}$  by finitely generated projective  $\mathbf{Z}\Gamma$ -modules.

All of the evidence thus far indicates the existence of a solution for the following

**CONJECTURE 65.1.** *With  $\mathfrak{G}(\mathcal{O}_S)$  as above and with  $k = \sum_{v \in S} \text{rank}_{K_v}(\mathfrak{G})$ , the arithmetic group  $\mathfrak{G}(\mathcal{O}_S)$  is of type  $\text{FP}_m$  if and only if either  $\text{rank}_K(\mathfrak{G}) = 0$  or  $k > m$ .*

For example, according to the above conjecture  $\text{SL}_n(\mathbf{F}_q[t])$  should be of type  $\text{FP}_{(n-2)}$  but not of type  $\text{FP}_{(n-1)}$ . In fact this was shown independently by Abels and Abramenko for large values of  $q$ .

Plenty of other evidence exists to support the conjecture in general including a proof of the “only if” implication [2]. The existing evidence for the

conjecture can be found in papers of Abels, Abramenko, Behr, Bux–Wortman, Hurrelbrink, Keller, Kneser, Lubotzky, McHardy, Nagao, O’Meara, Rehmann–Soulé, Serre, Splitthoff, and Stuhler. For detailed references, see e.g. [1] and [2].

## REFERENCES

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