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## 63

### A REALIZATION PROBLEM

by Kalathoor VARADARAJAN

In his groundbreaking papers ([7], [8]) C. T. C. Wall associated with each (always assumed 0-connected) finitely dominated space  $X$  an element  $\tilde{w}(X)$  in  $\widetilde{K}_0(\mathbf{Z}\pi)$  where  $\pi = \pi_1(X)$  and proved that  $X$  is of the homotopy type of a finite CW-complex if and only if  $\tilde{w}(X) = 0$ . Also  $\tilde{w}(X)$  is an invariant of the homotopy type of  $X$ . In subsequent literature  $\tilde{w}(X)$  is referred to as the finiteness obstruction (alternatively as the Wall obstruction) of  $X$ . Another major result proved by Wall asserts that given any finitely presented group  $\pi$  and any element  $x$  in  $\widetilde{K}_0(\mathbf{Z}\pi)$ , there exists a finitely dominated CW-complex  $X$  with  $\pi_1(X)$  isomorphic to  $\pi$  and  $\tilde{w}(X) = x$ . Using Dock Sang Rim's result [5] that  $\widetilde{K}_0(\mathbf{Z}\pi_p)$  for any prime  $p$  is isomorphic to the ideal class group  $\text{Cl}(\mathbf{Z}[\omega])$ , where  $\pi_p$  denotes a cyclic group of order  $p$  and  $\omega = \exp(\frac{2\pi i}{p})$  and the fact that  $\text{Cl}(\mathbf{Z}[\omega])$  is not zero when  $p = 23$ , Wall shows that there exist finitely dominated CW-complexes which are not of the homotopy type of a finite CW-complex. This settled a famous problem of J. H. C. Whitehead [9] in the negative.

Guido is the first person who started studying the Wall obstruction of finitely dominated nilpotent spaces [2] and [3]. In his 1976 work he proved that  $\tilde{w}(X) = 0$  for any finitely dominated nilpotent space with  $\pi_1(X)$  infinite. In his 1975 work he showed that if  $X$  is a finitely dominated nilpotent space with  $\pi_1(X)$  finite cyclic, then  $\tilde{w}(X)$  has to satisfy certain restrictions. Inspired by his results, I extended his 1975 results to finitely dominated nilpotent spaces with finite abelian fundamental groups. My result [6] appeared in 1978. For any nilpotent group  $\pi$ , let  $\overline{\mathbf{Z}\pi}$  denote a maximal order in  $\mathbf{Q}\pi$  containing  $\mathbf{Z}\pi$  and  $D(\mathbf{Z}\pi)$  denote the kernel of

$$j_* : \widetilde{K}_0(\mathbf{Z}\pi) \rightarrow \widetilde{K}_0(\overline{\mathbf{Z}\pi}).$$

In the joint paper [4] in 1979, Guido and myself showed that for any finitely dominated nilpotent space  $X$  with a finite (necessarily nilpotent) fundamental

group  $\pi$ , the Wall obstruction  $\tilde{w}(X)$  satisfies the restriction that  $\tilde{w}(X)$  is in  $D(\mathbf{Z}\pi)$ . This considerably strengthened the result in [6].

As stated earlier in this article, for any finitely presented group  $\pi$  and any element  $x$  in  $\widetilde{K}_0(\mathbf{Z}\pi)$ , there exists a finitely dominated CW-complex  $X$  with  $\pi_1(X) = \pi$  and  $\tilde{w}(X) = x$  (Wall's work in 1965, 1966). In [1], Ewing, Löffler and Pedersen showed that for a finite nilpotent group of composite order, the set of elements of  $\widetilde{K}_0(\mathbf{Z}\pi)$  that can be realized as the finiteness obstruction of a nilpotent space with fundamental group  $\pi$  is not in general equal to  $D(\mathbf{Z}\pi)$ . This suggests the following.

QUESTION 63.1. *Given a finite nilpotent group  $\pi$  characterize completely the elements in  $D(\mathbf{Z}\pi)$  which occur as the finiteness obstruction of a finitely dominated nilpotent space and for such an element  $x$  give an explicit construction of a finitely dominated nilpotent space  $X$  with  $\tilde{w}(X) = x$ .*

In this article, I have concentrated on just one aspect of Guido's work. His work is very profound and has influenced the development of topology in many ways.

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