

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 54 (2008)
Heft: 1-2

Artikel: On algebraic characterizations for the finiteness of the dimension of EG
Autor: Talelli, Olympia
DOI: <https://doi.org/10.5169/seals-109930>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 06.08.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

61

ON ALGEBRAIC CHARACTERIZATIONS FOR THE FINITENESS OF THE DIMENSION OF $\underline{E}G$

by Olympia TALELLI

In [5] the following theorem was proved :

THEOREM 61.1. *If G is an $\text{H}\mathfrak{F}$ -group of type FP_∞ then G admits a finite dimensional model for $\underline{E}G$.*

The class $\text{H}\mathfrak{F}$ was introduced by P. H. Kropholler in [4] and it is defined as the smallest class of groups containing the class of finite groups, with the property : if a group G admits a finite dimensional contractible G -CW-complex with all cell stabilizers in $\text{H}\mathfrak{F}$ then G is in $\text{H}\mathfrak{F}$.

This theorem, especially its proof, was the motivation for defining groups of type Φ in [7] and for proposing those as the ones which admit a finite dimensional model for $\underline{E}G$.

DEFINITION 61.2 ([7]). A group G is said to be of type Φ if it has the property that for every $\mathbf{Z}G$ -module M , $\text{projdim}_{\mathbf{Z}G} M < \infty$ if and only if $\text{projdim}_{\mathbf{Z}H} M < \infty$ for every finite subgroup H of G .

CONJECTURE 61.3. *The following statements are equivalent for a group G :*

- (1) *G admits a finite dimensional model for $\underline{E}G$.*
- (2) *G admits a finite dimensional contractible G -CW-complex with finite cell stabilizers.*
- (3) *G is of type Φ .*
- (4) *$\text{spli } \mathbf{Z}G < \infty$.*
- (5) *$\text{silp } \mathbf{Z}G < \infty$.*
- (6) *$\text{findim } \mathbf{Z}G < \infty$.*

The algebraic invariants $\text{spli } \mathbf{Z}G$ and $\text{silp } \mathbf{Z}G$ were defined in [3]: $\text{silp } \mathbf{Z}G$ is the supremum of the injective lengths of the projective $\mathbf{Z}G$ -modules and $\text{spli } \mathbf{Z}G$ is the supremum of the projective lengths of the injective $\mathbf{Z}G$ -modules. It was shown in [3] that $\text{silp } \mathbf{Z}G \leq \text{spli } \mathbf{Z}G$, and that if $\text{spli } \mathbf{Z}G < \infty$ then $\text{spli } \mathbf{Z}G = \text{silp } \mathbf{Z}G$. The *finitistic dimension* of $\mathbf{Z}G$, $\text{findim } \mathbf{Z}G$, is the supremum of the projective dimensions of the $\mathbf{Z}G$ -modules of finite projective dimension.

It is not very difficult to show that $(1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4) \Rightarrow (5) \Rightarrow (6)$, see [7].

If G is an HF -group of type FP_∞ then there is a bound on the orders of the finite subgroups of G [4], and $\text{projdim}_{\mathbf{Z}G} \mathbf{B}(G, \mathbf{Z})$ is finite, where $\mathbf{B}(G, \mathbf{Z})$ is the $\mathbf{Z}G$ -module of the bounded functions from G to \mathbf{Z} [1].

Theorem 61.1 is now an immediate consequence of the following theorem which was proved in [5]:

THEOREM 61.4. *If G is an HF -group such that there is a bound on the orders of the finite subgroups and $\text{projdim}_{\mathbf{Z}G} \mathbf{B}(G, \mathbf{Z})$ is finite, then G admits a finite dimensional model for $\underline{\mathbf{E}}G$.*

Since if G is an HF -group, $\text{projdim}_{\mathbf{Z}G} \mathbf{B}(G, \mathbf{Z})$ is finite if and only if $\text{findim } \mathbf{Z}G$ is finite [1], it follows from Theorem 61.4 that $(6) \Rightarrow (1)$ in Conjecture 61.3 if G is an HF -group with a bound on the orders of the finite subgroups. Moreover in [8] it is shown that $(6) \Rightarrow (1)$ if G is a torsion-free elementary amenable group.

In [6] it was shown that if a group G admits a finite dimensional model for $\underline{\mathbf{E}}G$ then for every finite subgroup H of G , $W(H)$ admits a finite dimensional model for $\underline{\mathbf{E}}W(H)$, where $W(H) = N_G(H)/H$. In [7] we show that if $\text{spli } \mathbf{Z}G$ is finite, then $\text{spli } \mathbf{Z}W(H)$ is finite for every finite subgroup H of G .

In support of Conjecture 61.3 is also the following characterization of finite groups, which we obtained in [2]: a group G is finite if and only if $\text{spli } \mathbf{Z}G = 1$.

REFERENCES

- [1] CORNICK, J. and P.H. KROPHOLLER. Homological finiteness conditions for modules over group algebras. *J. London Math. Soc.* (2) 58 (1998), 49–62.
- [2] DEMBEGIOTI, F. and O. TALELLI. An integral homological characterization of finite groups. *J. Algebra* 319 (2008), 267–271.

- [3] GEDRICH, T. V. and K. W. GRUENBERG. Complete cohomological functors on groups. *Topology Appl.* 25 (1987), 203–223.
- [4] KROPHOLLER, P. H. On groups of type FP_∞ . *J. Pure Appl. Algebra* 90 (1993), 55–67.
- [5] KROPHOLLER, P. H. and G. MISLIN. Groups acting on finite dimensional spaces with finite stabilizers. *Comment. Math. Helv.* 73 (1998), 122–136.
- [6] LÜCK, W. The type of the classifying space for a family of subgroups. *J. Pure Appl. Algebra* 149 (2000), 177–203.
- [7] TALELLI, O. On groups of type Φ . *Archiv der Math.* 89 (2007), 24–32.
- [8] TALELLI, O. A characterization for cohomological dimension for a big class of groups. (Preprint, 2007.)

Olympia Talelli

Department of Mathematics
University of Athens
Athens 15784
Greece
e-mail: otalelli@math.uoa.gr