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ON ALGEBRAIC CHARACTERIZATIONS FOR THE FINITENESS OF THE DIMENSION OF $\underline{E}G$

by Olympia TALELLI

In [5] the following theorem was proved :

THEOREM 61.1. *If G is an $\mathfrak{H}\mathfrak{F}$ -group of type FP_∞ then G admits a finite dimensional model for $\underline{E}G$.*

The class $\mathfrak{H}\mathfrak{F}$ was introduced by P.H. Kropholler in [4] and it is defined as the smallest class of groups containing the class of finite groups, with the property : if a group G admits a finite dimensional contractible G -CW-complex with all cell stabilizers in $\mathfrak{H}\mathfrak{F}$ then G is in $\mathfrak{H}\mathfrak{F}$.

This theorem, especially its proof, was the motivation for defining groups of type Φ in [7] and for proposing those as the ones which admit a finite dimensional model for $\underline{E}G$.

DEFINITION 61.2 ([7]). A group G is said to be of type Φ if it has the property that for every $\mathbf{Z}G$ -module M , $\text{projdim}_{\mathbf{Z}G} M < \infty$ if and only if $\text{projdim}_{\mathbf{Z}H} M < \infty$ for every finite subgroup H of G .

CONJECTURE 61.3. *The following statements are equivalent for a group G :*

- (1) G admits a finite dimensional model for $\underline{E}G$.
- (2) G admits a finite dimensional contractible G -CW-complex with finite cell stabilizers.
- (3) G is of type Φ .
- (4) $\text{spli } \mathbf{Z}G < \infty$.
- (5) $\text{silp } \mathbf{Z}G < \infty$.
- (6) $\text{findim } \mathbf{Z}G < \infty$.

The algebraic invariants $\text{spli } \mathbf{Z}G$ and $\text{silp } \mathbf{Z}G$ were defined in [3]: $\text{silp } \mathbf{Z}G$ is the supremum of the injective lengths of the projective $\mathbf{Z}G$ -modules and $\text{spli } \mathbf{Z}G$ is the supremum of the projective lengths of the injective $\mathbf{Z}G$ -modules. It was shown in [3] that $\text{silp } \mathbf{Z}G \leq \text{spli } \mathbf{Z}G$, and that if $\text{spli } \mathbf{Z}G < \infty$ then $\text{spli } \mathbf{Z}G = \text{silp } \mathbf{Z}G$. The *finitistic dimension* of $\mathbf{Z}G$, $\text{findim } \mathbf{Z}G$, is the supremum of the projective dimensions of the $\mathbf{Z}G$ -modules of finite projective dimension.

It is not very difficult to show that $(1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4) \Rightarrow (5) \Rightarrow (6)$, see [7].

If G is an $\mathfrak{H}\mathfrak{F}$ -group of type FP_∞ then there is a bound on the orders of the finite subgroups of G [4], and $\text{projdim}_{\mathbf{Z}G} B(G, \mathbf{Z})$ is finite, where $B(G, \mathbf{Z})$ is the $\mathbf{Z}G$ -module of the bounded functions from G to \mathbf{Z} [1].

Theorem 61.1 is now an immediate consequence of the following theorem which was proved in [5]:

THEOREM 61.4. *If G is an $\mathfrak{H}\mathfrak{F}$ -group such that there is a bound on the orders of the finite subgroups and $\text{projdim}_{\mathbf{Z}G} B(G, \mathbf{Z})$ is finite, then G admits a finite dimensional model for $\underline{E}G$.*

Since if G is an $\mathfrak{H}\mathfrak{F}$ -group, $\text{projdim}_{\mathbf{Z}G} B(G, \mathbf{Z})$ is finite if and only if $\text{findim } \mathbf{Z}G$ is finite [1], it follows from Theorem 61.4 that $(6) \Rightarrow (1)$ in Conjecture 61.3 if G is an $\mathfrak{H}\mathfrak{F}$ -group with a bound on the orders of the finite subgroups. Moreover in [8] it is shown that $(6) \Rightarrow (1)$ if G is a torsion-free elementary amenable group.

In [6] it was shown that if a group G admits a finite dimensional model for $\underline{E}G$ then for every finite subgroup H of G , $W(H)$ admits a finite dimensional model for $\underline{E}W(H)$, where $W(H) = N_G(H)/H$. In [7] we show that if $\text{spli } \mathbf{Z}G$ is finite, then $\text{spli } \mathbf{Z}W(H)$ is finite for every finite subgroup H of G .

In support of Conjecture 61.3 is also the following characterization of finite groups, which we obtained in [2]: a group G is finite if and only if $\text{spli } \mathbf{Z}G = 1$.

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