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CLASSIFYING SPACES FOR PROPER ACTIONS

by Peter H. KROPHOLLER

It is an open problem to find good algebraic criteria for a group G to admit a finite dimensional model for $\underline{E}G$, the classifying space for proper actions. Recall that this classifying space is a proper G-CW-complex such that every finite subgroup has contractible fixed-point set and it is uniquely determined up to G-homotopy equivalence. Guido Mislin introduced me to this concept some twelve years ago and together [2] we proved a theorem about it: namely that every $\mathrm{H}\mathfrak{F}$-group of type FP_∞ has a finite dimensional $\underline{E}$. This theorem was an improvement of my original conjecture that $\mathrm{H}\mathfrak{F}$-groups of type FP_∞ should belong to $\mathrm{H}_1\mathfrak{F}$. The class $\mathrm{H}_1\mathfrak{F}$ consists of all groups which admit a proper action on a finite dimensional contractible CW-complex. The proof of the Kropholler–Mislin theorem applies to a wider class than the FP_∞ groups, showing that all $\mathrm{H}_1\mathfrak{F}$-groups for which there is a bound on the orders of the finite subgroups have finite dimensional models for $\underline{E}$. Therefore there is the following natural conjecture.

CONJECTURE 45.1. *Every $\mathrm{H}_1\mathfrak{F}$-group has a finite dimensional classifying space for proper actions.*

Examples which do not fall within the scope of the Kropholler–Mislin method include quasicyclic groups, the lamplighter group and many others. However, in all known cases of such examples it is always possible to verify the conjecture very easily. The conjecture is of theoretical interest and remains tantalizing.

Wolfgang Lück introduced [3] the use of the orbit category in this context and greatly improved the dimension bounds on proper classifying spaces, as can be seen in Theorem 6.4 of [3]. Lück's work provides an algebraic framework for research in this area. Subsequently others have contributed:

notable examples include the work [1] of Brady–Leary–Nucinkis and the studies [4], [5], [6] made by Martínez-Pérez and Nucinkis. Nevertheless there is at present no proof or refutation of the above conjecture.

REFERENCES

[1] BRADY, N., I. J. LEARY and B. E. A. NUCINKIS. On algebraic and geometric dimensions for groups with torsion. *J. London Math. Soc. (2) 64* (2001), 489–500.

[2] KROPHOLLER, P. H. and G. MISLIN. Groups acting on finite-dimensional spaces with finite stabilizers. *Comment. Math. Helv. 73* (1998), 122–136.

[3] LÜCK, W. The type of the classifying space for a family of subgroups. *J. Pure Appl. Algebra 149* (2000), 177–203.

[4] MARTÍNEZ-PÉREZ, C. A spectral sequence in Bredon (co)homology. *J. Pure Appl. Algebra 176* (2002), 161–173.

[5] MARTÍNEZ-PÉREZ, C. and B. E. A. NUCINKIS. Cohomological dimension of Mackey functors for infinite groups. *J. London Math. Soc. (2) 74* (2006), 379–396.

[6] NUCINKIS, B. E. A. On dimensions in Bredon homology. *Homology Homotopy Appl. 6* (2004), 33–47.

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