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## 41

### COHOMOLOGY OF PURE AUTOMORPHISM GROUPS OF FREE PRODUCTS OF FINITE GROUPS

by Craig JENSEN

Recently, John McCammond, John Meier and I verified [2] the Brownstein–Lee conjecture [1]. It concerns the cohomology of the *pure symmetric automorphism group of a free group*,  $P\Sigma_n$ , namely the group which has to take each of the preferred generators of free group  $F_n$  to a conjugate of itself.

THEOREM 41.1 (The Brownstein–Lee Conjecture). *The cohomology ring  $H^*(P\Sigma_n, \mathbf{Z})$  is generated by one-dimensional classes  $\alpha_{ij}^*$ , where  $i \neq j$ , subject to the relations*

- (1)  $\alpha_{ij}^* \wedge \alpha_{ij}^* = 0$
- (2)  $\alpha_{ij}^* \wedge \alpha_{ji}^* = 0$
- (3)  $\alpha_{kj}^* \wedge \alpha_{ji}^* = (\alpha_{kj}^* - \alpha_{ij}^*) \wedge \alpha_{ki}^*$

and the Poincaré series is  $p(z) = (1 + nz)^{n-1}$ .

What can we say about the cohomology of similar groups ? A first question might be :

QUESTION 41.2. *What is the cohomology of  $PAut(\mathbf{Z}/p * \cdots * \mathbf{Z}/p)$  ? That is, what is the cohomology of the pure (meaning each  $\mathbf{Z}/p$  in the free product has to be taken to a conjugate of itself) automorphism group of a free product of  $n$  copies of  $\mathbf{Z}/p$  ?*

After we get this, a more generalized question would be :

QUESTION 41.3. *Let  $G_1, \dots, G_n$  be finite abelian groups. What is the cohomology of  $PAut(G_1 * \cdots * G_n)$  ?*

or perhaps

QUESTION 41.4. *Let  $G_1, \dots, G_n$  be finite abelian groups or  $\mathbf{Z}$ . What is the cohomology of  $PAut(G_1 * \dots * G_n)$ ?*

or perhaps even

QUESTION 41.5. *Let  $G_1, \dots, G_n$  be finite groups or  $\mathbf{Z}$ . What is the cohomology of  $PAut(G_1 * \dots * G_n)$ ?*

#### REFERENCES

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