Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 54 (2008)
Heft: 1-2

Artikel: Planar 2-cocycles of finite groups

Autor: Chen, Yu Qing

DOI: https://doi.org/10.5169/seals-109893

L'Enseignement Mathématique (2) 54 (2008), 73–74

24

PLANAR 2-COCYCLES OF FINITE GROUPS

by Yu Qing CHEN

Let G be a finite group and A a G-module. Recall [2] that a (normalized) 2-cocycle of G with coefficients in A is a function

$$f\colon G \times G \to A$$

satisfying

(i) $f(g,1) = f(1,g) = 0$, for all $g \in G$;

(ii) $f(g,h) + f(gh,k) = gf(h,k) + f(g,hk)$ for all $g,h,k \in G$.

DEFINITION 24.1. A 2-cocycle of G with coefficients in A is called *planar* (the extension group acts on a finite projective plane as a collineation group [3], [4], [5]) if

(i) $|G| = |A|$;

(ii) for every $1 \neq g \in G$, the maps

$$f(g,\ )\colon G \to A$$

and

$$f(\ ,g)\colon G \to A$$

are bijections.

EXAMPLE 24.2. Let $\mathbf{F}$ be a finite field. We can regard $\mathbf{F}$ as a trivial $\mathbf{F}$-module. For any Galois automorphism σ, we define $f_\sigma\colon \mathbf{F} \times \mathbf{F} \to \mathbf{F}$ by

$$f_\sigma(x,y) = x\,\sigma(y)\,.$$

The function f_σ is a planar 2-cocycle of the additive group of $\mathbf{F}$ with coefficients in the same group.

CONJECTURE 24.3. *If G has a planar 2-cocycle, then G is a p-group.*

REMARK 24.4. A more general conjecture is the so-called prime power conjecture, which asserts that the order of a finite projective plane is always a power of a prime ([1], [5]).

CONJECTURE 24.5 (Stronger version). *If G has a planar 2-cocycle with coefficients in a G-module A, then G and the underlying group of A are elementary abelian groups.*

REMARK 24.6. In the case when G is abelian and the cocycle is symmetric (i.e. $f(x,y) = f(y,x)$ for all x and y in G), which is equivalent to the extension group being abelian, Conjecture 24.3 is true [1] but we do not know whether the stronger version is also true.

REFERENCES

[1] BLOKHUIS, A., D. JUNGNICKEL and B. SCHMIDT. Proof of the prime power conjecture for projective planes of order n with abelian collineation groups of order n^2. *Proc. Amer. Math. Soc. 130* (2002), 1473–1476.

[2] BROWN, K. S. *Cohomology of Groups*. Graduate Texts in Mathematics *87*. Springer-Verlag, Berlin, Heidelberg, New York, 1982.

[3] GALATI, J. C. A group extension approach to relative difference sets. *J. Combin. Des. 12* (2004), 279–298.

[4] PERERA, A. A. I. and K. J. HORADAM. Cocyclic generalised Hadamard matrices and central relative difference sets. *Des. Codes Cryptogr. 15* (1998), 187–200.

[5] POTT, A. *Finite Geometry and Character Theory*. Lecture Notes in Mathematics *1601*. Springer-Verlag, Berlin, Heidelberg, 1995.

Yu Qing Chen

Department of Mathematics and Statistics
Wright State University
Dayton, OH 45435
USA
e-mail: yuqing.chen@wright.edu