

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 54 (2008)  
**Heft:** 1-2

**Artikel:** Planar 2-cocycles of finite groups  
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**DOI:** <https://doi.org/10.5169/seals-109893>

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## 24

### PLANAR 2-COCYCLES OF FINITE GROUPS

by Yu Qing CHEN

Let  $G$  be a finite group and  $A$  a  $G$ -module. Recall [2] that a (normalized) 2-cocycle of  $G$  with coefficients in  $A$  is a function

$$f: G \times G \rightarrow A$$

satisfying

- (i)  $f(g, 1) = f(1, g) = 0$ , for all  $g \in G$ ;
- (ii)  $f(g, h) + f(gh, k) = gf(h, k) + f(g, hk)$  for all  $g, h, k \in G$ .

DEFINITION 24.1. A 2-cocycle of  $G$  with coefficients in  $A$  is called *planar* (the extension group acts on a finite projective plane as a collineation group [3], [4], [5]) if

- (i)  $|G| = |A|$ ;
- (ii) for every  $1 \neq g \in G$ , the maps

$$f(g, \ ): G \rightarrow A$$

and

$$f(\ , g): G \rightarrow A$$

are bijections.

EXAMPLE 24.2. Let  $\mathbf{F}$  be a finite field. We can regard  $\mathbf{F}$  as a trivial  $\mathbf{F}$ -module. For any Galois automorphism  $\sigma$ , we define  $f_\sigma: \mathbf{F} \times \mathbf{F} \rightarrow \mathbf{F}$  by

$$f_\sigma(x, y) = x\sigma(y).$$

The function  $f_\sigma$  is a planar 2-cocycle of the additive group of  $\mathbf{F}$  with coefficients in the same group.

CONJECTURE 24.3. *If  $G$  has a planar 2-cocycle, then  $G$  is a  $p$ -group.*

REMARK 24.4. A more general conjecture is the so-called prime power conjecture, which asserts that the order of a finite projective plane is always a power of a prime ([1], [5]).

CONJECTURE 24.5 (Stronger version). *If  $G$  has a planar 2-cocycle with coefficients in a  $G$ -module  $A$ , then  $G$  and the underlying group of  $A$  are elementary abelian groups.*

REMARK 24.6. In the case when  $G$  is abelian and the cocycle is symmetric (i.e.  $f(x, y) = f(y, x)$  for all  $x$  and  $y$  in  $G$ ), which is equivalent to the extension group being abelian, Conjecture 24.3 is true [1] but we do not know whether the stronger version is also true.

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