

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 54 (2008)  
**Heft:** 1-2

**Artikel:** Planar 2-cocycles of finite groups  
**Autor:** Chen, Yu Qing  
**DOI:** <https://doi.org/10.5169/seals-109893>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 22.11.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## 24

### PLANAR 2-COCYCLES OF FINITE GROUPS

by Yu Qing CHEN

Let  $G$  be a finite group and  $A$  a  $G$ -module. Recall [2] that a (normalized) 2-cocycle of  $G$  with coefficients in  $A$  is a function

$$f: G \times G \rightarrow A$$

satisfying

- (i)  $f(g, 1) = f(1, g) = 0$ , for all  $g \in G$ ;
- (ii)  $f(g, h) + f(gh, k) = gf(h, k) + f(g, hk)$  for all  $g, h, k \in G$ .

DEFINITION 24.1. A 2-cocycle of  $G$  with coefficients in  $A$  is called *planar* (the extension group acts on a finite projective plane as a collineation group [3], [4], [5]) if

- (i)  $|G| = |A|$ ;
- (ii) for every  $1 \neq g \in G$ , the maps

$$f(g, \ ): G \rightarrow A$$

and

$$f(\ , g): G \rightarrow A$$

are bijections.

EXAMPLE 24.2. Let  $\mathbf{F}$  be a finite field. We can regard  $\mathbf{F}$  as a trivial  $\mathbf{F}$ -module. For any Galois automorphism  $\sigma$ , we define  $f_\sigma: \mathbf{F} \times \mathbf{F} \rightarrow \mathbf{F}$  by

$$f_\sigma(x, y) = x\sigma(y).$$

The function  $f_\sigma$  is a planar 2-cocycle of the additive group of  $\mathbf{F}$  with coefficients in the same group.

CONJECTURE 24.3. *If  $G$  has a planar 2-cocycle, then  $G$  is a  $p$ -group.*

REMARK 24.4. A more general conjecture is the so-called prime power conjecture, which asserts that the order of a finite projective plane is always a power of a prime ([1], [5]).

CONJECTURE 24.5 (Stronger version). *If  $G$  has a planar 2-cocycle with coefficients in a  $G$ -module  $A$ , then  $G$  and the underlying group of  $A$  are elementary abelian groups.*

REMARK 24.6. In the case when  $G$  is abelian and the cocycle is symmetric (i.e.  $f(x, y) = f(y, x)$  for all  $x$  and  $y$  in  $G$ ), which is equivalent to the extension group being abelian, Conjecture 24.3 is true [1] but we do not know whether the stronger version is also true.

#### REFERENCES

- [1] BLOKHUIS, A., D. JUNGnickEL and B. SCHMIDT. Proof of the prime power conjecture for projective planes of order  $n$  with abelian collineation groups of order  $n^2$ . *Proc. Amer. Math. Soc.* 130 (2002), 1473–1476.
- [2] BROWN, K. S. *Cohomology of Groups*. Graduate Texts in Mathematics 87. Springer-Verlag, Berlin, Heidelberg, New York, 1982.
- [3] GALATI, J. C. A group extension approach to relative difference sets. *J. Combin. Des.* 12 (2004), 279–298.
- [4] PERERA, A. A. I. and K. J. HORADAM. Cocyclic generalised Hadamard matrices and central relative difference sets. *Des. Codes Cryptogr.* 15 (1998), 187–200.
- [5] POTT, A. *Finite Geometry and Character Theory*. Lecture Notes in Mathematics 1601. Springer-Verlag, Berlin, Heidelberg, 1995.

Yu Qing Chen

Department of Mathematics and Statistics  
Wright State University  
Dayton, OH 45435  
USA  
*e-mail*: yuqing.chen@wright.edu