

# Automorphism groups of right-angled Artin groups

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### AUTOMORPHISM GROUPS OF RIGHT-ANGLED ARTIN GROUPS

by Ruth CHARNEY and Karen VOGTMANN

We propose some problems concerning outer automorphism groups of right-angled Artin groups (“RAAG’s”). Since  $F_n$  and  $\mathbf{Z}^n$  are examples of RAAG’s, it is tempting to view outer automorphism groups of general right-angled Artin groups as interpolating between  $Out(F_n)$  and  $GL(n, \mathbf{Z})$ , and to ask to what extent properties common to both  $Out(F_n)$  and  $GL(n, \mathbf{Z})$  are shared by all of these groups.

Recall that a RAAG  $A_\Gamma$  based on a simplicial graph  $\Gamma$  is generated by the vertices of  $\Gamma$ , and the only relations are that  $v$  commutes with  $w$  if  $v$  and  $w$  are joined by an edge of  $\Gamma$ . Servatius [4] and Laurence [3] gave generators for  $Aut(A_\Gamma)$ , but not much else is known about this group except in the cases when  $\Gamma$  has no edges (so  $A_\Gamma$  is free) and  $\Gamma$  is the complete graph (so  $A_\Gamma$  is free abelian). Laurence’s generators are of four types: (1) inner automorphisms, (2) symmetries of  $\Gamma$  and inversions of the generators  $v$ , (3) *partial conjugations*, which conjugate everything in some connected component of  $\Gamma - st(v)$  by  $v$ , and (4) *transvections*, which multiply  $v$  by  $w$  (on the right or left) if  $lk(v) \subseteq st(w)$ . Thus every element of  $Out(A_\Gamma)$  lifts to an automorphism of the free group on the vertices of  $\Gamma$ , but the natural map from  $Out(A_\Gamma)$  to  $GL(n, \mathbf{Z})$  is usually not surjective.

There is a CAT(0) cube complex associated to any RAAG, whose 1-skeleton is the Cayley graph of the group, and which has a  $k$ -dimensional cube whenever the 1-skeleton of the cube appears (kind of a “cube-flag” complex, see [2]). The RAAG acts freely on this; the quotient has a loop for each generator and a  $k$ -torus for each complete subgraph on  $k$  vertices in  $\Gamma$ . This cube complex is 2-dimensional if and only if the graph  $\Gamma$  has no triangles. In this case, we have constructed an “outer space” for  $Out(A_\Gamma)$ , which is a contractible space on which  $Out(A_\Gamma)$  acts with finite stabilizers [1]. Points in this outer space are morally actions of  $A_\Gamma$  on 2-dimensional CAT(0) complexes,

though the actual description is in terms of products of trees. This outer space is finite-dimensional, and we obtain :

**THEOREM 23.1.** *If  $\Gamma$  is connected and triangle-free then  $\text{Out}(A_\Gamma)$  has a torsionfree subgroup of finite index and it has finite virtual cohomological dimension.*

Although we obtain both upper and lower bounds on the virtual cohomological dimension, these bounds in general do not match, so we ask :

**QUESTION 23.2.** *What is the exact virtual cohomological dimension of the outer automorphism group of a 2-dimensional right-angled Artin group ?*

The no-triangles condition on  $\Gamma$  was very convenient, but of course we would like to know what happens for any  $\Gamma$  :

**QUESTION 23.3.** *Do the outer automorphism groups of all right-angled Artin groups have torsion-free subgroups of finite index ?*

**QUESTION 23.4.** *Calculate the virtual cohomological dimension of the automorphism group of any right-angled Artin group.*

ADDED IN PROOF. For progresses on these problems see : R. CHARNEY and K. VOGTMANN. ‘Finiteness properties of automorphism groups of right-angled Artin groups’, submitted. And also : BUX, K.-U., R. CHARNEY and K. VOGTMANN. ‘Automorphisms of two-dimensional RAAGS and partially symmetric automorphisms of free groups’, submitted.

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