

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 53 (2007)  
**Heft:** 1-2

**Artikel:** Using Gauss maps to detect intersections

**Autor:** Xavier, Frederico

### Bibliographie

**DOI:** <https://doi.org/10.5169/seals-109537>

### Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 06.08.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

intervals. Here one should note that the solutions of (8) are independent of the choice of frames, since they uniquely solve (6). Furthermore, the sets  $\mathcal{O}_p$  are pairwise disjoint, by the uniqueness of solutions of initial value problems for ordinary differential equations with smooth coefficients. Hence,

$$M_1 \times \cdots \times M_k = \bigcup_{p \in f_1(M_1) \cap \cdots \cap f_k(M_k)} \mathcal{O}_p,$$

where, as observed, the sets  $\mathcal{O}_p$  are non-empty, open and pairwise disjoint. Connectedness of  $M_1 \times \cdots \times M_k$  now implies that there is only one such set, thus showing that  $f_1(M_1) \cap \cdots \cap f_k(M_k)$  reduces to a single point. This concludes the proof of Theorem 3.

#### REFERENCES

- [1] BALREIRA, E. Detecting invertibility from the topology of the pre-images of hyperplanes. Doctoral Dissertation, University of Notre Dame (2006).
- [2] BASS, H., E. CONNELL and D. WRIGHT. The Jacobian conjecture: reduction of the degree and formal expansion of the inverse. *Bull. Amer. Math. Soc.* 7 (1982), 287–330.
- [3] VAN DEN ESSEN, A. Polynomial Automorphisms and the Jacobian Conjecture. *Progress in Mathematics* 190, Birkhäuser (2000).
- [4] JUNG, H.W.E. Über ganze birationale Transformationen der Ebene. *J. Reine Angew. Math.* 184 (1942), 161–174.
- [5] KATOK, A. and B. HASSELBLATT. Introduction to the Modern Theory of Dynamical Systems. *Encyclopedia of Mathematics and Applications*, vol. 54. Cambridge University Press (1995).
- [6] KULIKOV, V. Generalized and local Jacobian problems. *Russian Acad. Sci. Izv. Math.* 41 (1993), 351–365.
- [7] NOLLET, S. and F. XAVIER. Global inversion via the Palais-Smale condition. *Contin. Dyn. Syst.* 8 (2002), 17–28.
- [8] —— Holomorphic injectivity and the Hopf map. *Geom. Funct. Anal.* 14 (2004), 1339–1351.
- [9] —— On Kulikov’s problem. To appear in *Archiv der Mathematik*.
- [10] NOLLET, S., L. TAYLOR and F. XAVIER. Birationality of étale maps via surgery. (Preprint.)
- [11] PINCHUK, S. A Counterexample to the strong real Jacobian conjecture. *Math. Z.* 217 (1994), 1–4.
- [12] RABIER, P.J. Ehresmann fibrations and Palais-Smale conditions for morphisms of Finsler manifolds. *Ann. of Math.* (2) 146 (1997), 647–691.

- [13] XAVIER, F. Injectivity as a transversality phenomenon in geometries of negative curvature. *Illinois J. Math.* 43 (1999), 256–263.
- [14] —— Rigidity of the identity. To appear in *Communications in Contemporary Mathematics*.

(Reçu le 24 mai 2006)

Frederico Xavier

Department of Mathematics  
University of Notre Dame  
Notre Dame IN 46556  
U. S. A.  
*e-mail* : xavier.1@nd.edu