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where (F_i, G_j) in a $2n$-tuple of smooth functions on M. The space TVect(M) is now identified with the direct sum

$$\mathrm{TVect}(M) \cong \underbrace{C^\infty(M) \oplus \cdots \oplus C^\infty(M)}_{2n \text{ times}} .$$

Let us calculate explicitly the action of CVect(M) on TVect(M).

PROPOSITION 5.4. *The action of* CVect(M) *on* TVect(M) *is given by the first-order* $(2n \times 2n)$*-matrix differential operator*

$$X_H \begin{pmatrix} F \\ G \end{pmatrix} = \left(X_H \cdot \mathbf{1} - \begin{pmatrix} AB(H) & BB(H) \\ -AA(H) & -BA(H) \end{pmatrix} \right) \begin{pmatrix} F \\ G \end{pmatrix}, \tag{14}$$

where F and G are n-vector functions, $\mathbf{1}$ *is the unit* $(2n \times 2n)$*-matrix,* $AA(H)$, $AB(H)$, $BA(H)$ *and* $BB(H)$ *are* $(n \times n)$*-matrices, namely*

$$AA(H)_{ij} = A_i A_j(H),$$

and the three other expressions are similar.

Proof. Straightforward from (11) and (13). □

PROPOSITION 5.5. *The bilinear map (7) has the following explicit expression:*

$$H_{X,\widetilde{X}} = \sum_{i=1}^{n} \begin{vmatrix} F_i & \widetilde{F}_i \\ G_i & \widetilde{G}_i \end{vmatrix},$$

where $X = \sum_{i=1}^{n} (F_i A_i + G_i B_i)$, *and* $\widetilde{X} = \sum_{j=1}^{n} (\widetilde{F}_j A_j + \widetilde{G}_j B_j)$.

Proof. This follows from definition (7) and formula (12). □

Note that formula (14) implies that $H_{X,\widetilde{X}}$ transforms as a contact Hamiltonian according to (3) since the partial traces of the $(2n \times 2n)$-matrix in (14) are $A_i B_i(H) - B_i A_i(H) = Z(H)$.

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