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where

$$\tilde{\Omega}_n = \begin{pmatrix} 1 & & 0 & \dots & 0 \\ & \ddots & & & \\ 1-t & & 1 & & \vdots \\ \vdots & & & \ddots & 0 \\ 1-t & \dots & 1-t & & 1 \end{pmatrix}$$

REMARK. Squier [Sq] gives an “hermitian” matrix M_n such that

$$\overline{Bu(\sigma)}^t \times M_n \times Bu(\sigma) = M_n,$$

but our matrix $\tilde{\Omega}_n$ is much simpler for two reasons :

- (a) $\tilde{\Omega}_n \in GL_n(\mathbf{Z}[t, t^{-1}])$, whereas $M_n \in GL_n(\mathbf{Z}[t^{\pm 1/2}])$;
- (b) $\tilde{\Omega}_n$ is triangular.

The fact that $\tilde{\Omega}_n$ is triangular imposes more constraints on a matrix to be a Burau matrix, than that of Squier. This will help to understand the group of Burau matrices (recall that we know that the Burau representation is not faithful for $n \geq 5$ by [Moo], [L; P], [Bg]).

COROLLARY 5.4. *Corollary 5.2 is true, if Gassner matrices are replaced by Burau matrices.*

Added in proof. After this paper had been written, the author was informed (in June 2005) that Theorem 0.1 and Lemma 1.2 were obtained previously by V. Turaev in a paper “Intersection loops in two-dimensional manifolds”, which appeared in *Mathematics of the USSR Sbornik* 35 (1979).

REFERENCES

- [Bg] BIGELOW, S. The Burau representation is not faithful for $n = 5$. *Geom. Topol.* 3 (1999), 397–404.
- [Bi] BIRMAN, J. *Braids, Links and Mapping Class Groups*. Ann. of Math. Stud. 82. Princeton University Press, 1974.
- [F] FOX, R. Free differential calculus I. *Ann. of Math.* (2) 57 (1953), 547–560.
- [H] HEMPEL, J. Intersection calculus on surfaces with applications to 3-manifolds. *Mem. Amer. Math. Soc.* 43, 282 (1983).
- [LP] LONG, D.D. and M. PATON. The Burau representation is not faithful for $n \geq 6$. *Topology* 32 (1993), 439–447.

- [Moo] MOODY, J. The Burau representation of the braid group B_n is unfaithful for large n . *Bull. Amer. Math. Soc. (N.S.)* 25 (1991), 379–384.
- [Mo₁] MORITA, S. Abelian quotients of subgroups of the mapping class group of surfaces. *Duke Math J.* 70 (1993), 699–725.
- [Mo₂] —— Structure of the mapping class group of surfaces: a survey and a prospect. *Geom. Topol. Monogr.* 2 (1999), 349–406.
- [Pap] PAPAKYRIAKOPOULOS, C. D. Planar regular coverings of orientable, closed Surfaces. *Knots, Groups and 3-Manifolds*. Ann. of Math. Stud. 84, Princeton University Press, 1975, 261–292.
- [PaR] PARIS, L. and D. ROLFSEN. Geometric subgroups of mapping class groups. *J. reine angew. Math.* 521 (2000), 47–83.
- [Pe] PERRON, B. Homomorphic extensions of Johnson homomorphisms via Fox calculus. *Ann. Inst. Fourier (Grenoble)* 54 (2004), 1073–1106.
- [Sq] SQUIER, C. The Burau representation is unitary. *Proc. Amer. Math. Soc.* 90 (1984), 199–202.
- [Su] SUZUKI, M. The Magnus representation of the Torelli group $I_{g,1}$ is not faithful for $g \geq 2$. *Proc. Amer. Math. Soc.* 130 (2002), 909–914.

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