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(3) Let H be a closed subgroup of a topological group G . Say that G has *Property (T)* relative to H if any unitary representation π of G which almost has invariant vectors and which has non-zero invariant vectors by H has also non-zero vectors invariant by G . (In particular, Property (T) for the group G itself means that G has Property (T) relative to $\{1\}$.) This property plays its role in Popa's articles [14] and [17], but it should not be confused with Property (T) for the pair $H \subset G$.

Added in proof. 1) Using the recent article of J. Peterson, A 1-cohomology characterization of property (T) in von Neumann algebras, preprint 2004, it can be proved that condition (a4) in Theorem 1.2 implies condition (b1) when G is a σ -compact group.

2) We owe to Y. de Cornulier the following two observations: First, it is unnecessary to assume that G is Hausdorff since every unitary representation is trivial on the closure of $\{1\}$. Secondly, statement (1) in Corollary 4.1 can be deduced from Valette's property (FH): see Proposition 2.5.5 in B. Bekka, P. de la Harpe and A. Valette, Kazhdan's Property (T), preprint 2003.

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