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(3) Let H be a closed subgroup of a topological group G . Say that G has *Property (T) relative to H* if any unitary representation π of G which almost has invariant vectors and which has non-zero invariant vectors by H has also non-zero vectors invariant by G . (In particular, Property (T) for the group G itself means that G has Property (T) relative to $\{1\}$.) This property plays its role in Popa's articles [14] and [17], but *it should not* be confused with Property (T) for the pair $H \subset G$.

Added in proof. 1) Using the recent article of J. Peterson, A 1-cohomology characterization of property (T) in von Neumann algebras, preprint 2004, it can be proved that condition (a4) in Theorem 1.2 implies condition (b1) when G is a σ -compact group.

2) We owe to Y. de Cornulier the following two observations: First, it is unnecessary to assume that G is Hausdorff since every unitary representation is trivial on the closure of $\{1\}$. Secondly, statement (1) in Corollary 4.1 can be deduced from Valette's property (FH): see Proposition 2.5.5 in B. Bekka, P. de la Harpe and A. Valette, Kazhdan's Property (T), preprint 2003.

REFERENCES

- [1] AKEMANN, C. A. and M. E. WALTER. Unbounded negative definite functions. *Canad. J. Math.* 33 (1981), 862–871.
- [2] BATES, T. and A. G. ROBERTSON. Positive definite functions and relative Property (T) for subgroups of discrete groups. *Bull. Austral. Math. Soc.* 52 (1995), 31–39.
- [3] BEKKA, B. Kazhdan's Property (T) for the unitary group of a separable Hilbert space. *Geom. Funct. Anal.* 13 (2003), 509–520.
- [4] CHERIX, P.-A., M. COWLING, P. JOLISSAINT, P. JULG and A. VALETTE. *Groups with the Haagerup property (Gromov's a -T-menability)*. *Progress in Mathematics* 197. Birkhäuser (Basel), 2001.
- [5] DE LA HARPE, P. and A. VALETTE. *La propriété (T) de Kazhdan pour les groupes localement compacts*. *Astérisque* 175, 1989.
- [6] EYMARD, P. Moyennes invariantes et représentations unitaires. *Lecture Notes in Mathematics* 300. Springer (New York), 1972.
- [7] GAAL, S. *Linear Analysis and Representation Theory*. Springer (New York), 1973.
- [8] JOLISSAINT, P. Property T for discrete groups in terms of their regular representation. *Math. Ann.* 297 (1993), 539–551.
- [9] ——— Borel cocycles, approximation properties and relative Property T. *Ergodic Theory Dynam. Systems* 20 (2000), 483–499.
- [10] KAZHDAN, D. Connection of the dual space of a group with the structure of its closed subgroups. *Funct. Anal. Appl.* 1 (1967), 63–65.

- [11] MARGULIS, G. A. Explicit constructions of concentrators. *Problems Inform. Transmission* 9 (1973), 325–332.
- [12] — Finitely-additive invariant measures on Euclidean spaces. *Ergodic Theory Dynam. Systems* 2 (1982), 383–396.
- [13] NICOARA, R., S. POPA and R. SASYK. *Some remarks on irrational rotation HT factors*. Preprint, 2004.
- [14] POPA, S. *Correspondences*. Unpublished, 1986.
- [15] — *On a class of type II_1 factors with Betti numbers invariants*. MSRI preprint, 2001-005.
- [16] — *Some rigidity results for non-commutative Bernoulli shifts*. MSRI preprint, 2001-024, revised version math. OA/0209130.
- [17] — Some properties of the symmetric enveloping algebra of a subfactor, with applications to amenability and property T. *Doc. Math.* 4 (1999), 665–744.
- [18] SHALOM, Y. Bounded generation and Kazhdan’s Property (T). *Inst. Hautes Études Sci. Publ. Math.* 90 (1999), 145–168.
- [19] ZIMMER, R. *Ergodic Theory and Semisimple Groups*. Birkhäuser (Basel), 1984.

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