**Zeitschrift:** L'Enseignement Mathématique
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique
**Band:** 51 (2005)
**Heft:** 3-4: L'enseignement mathématique

**Artikel:** A report on the ICMI study: "The role of the history of mathematics in the teaching and learning of mathematics"
**Autor:** Furinghetti, Fulvia
**DOI:** https://doi.org/10.5169/seals-3604

*L'Enseignement Mathématique, t. 51 (2005), p. 365–372*

# COMMISSION INTERNATIONALE DE L'ENSEIGNEMENT MATHÉMATIQUE

# (THE INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION)

## A REPORT ON THE ICMI STUDY: "THE ROLE OF THE HISTORY OF MATHEMATICS IN THE TEACHING AND LEARNING OF MATHEMATICS"

by Fulvia FURINGHETTI *)

### 1. ICMI STUDY 10

ICMI Study 10 is entitled "The role of the history of mathematics in the teaching and learning of mathematics". The discussion document [4] initially appeared in 1997; the Study Conference took place at CIRM, the country retreat of the French Mathematical Society at Luminy (near Marseille, France) from 20 to 25 April 1998. The volume resulting from the Study and edited by the two co-chairs [9] was presented at ICME 9 in Japan (2000). The argument of this ICMI Study is far more wide-ranging than discussing the opportunity of teaching some elements of the history of mathematics in the mathematics courses of the various school levels and at university: it concerns the goals of mathematics teaching and the way to reach them through history. As the editors of the Study volume put it (p. xvii), the ICMI Study "is posited on the experience of many mathematics teachers across the world that its history makes a difference: that having the history of mathematics as a resource for the teacher is beneficial. School mathematics reflects the wider aspect of mathematics as a cultural activity."

*) Thanks to Peter Ransom for helping to revise the English of this paper.

The theme of ICMI Study 10 had never been considered in previous ICMI Studies, but the relation between mathematics and its history has interested the world of mathematics education for a long time. This happened especially around the end of the nineteenth and the beginning of the twentieth century. At those times the history of mathematics was developing as an autonomous discipline: journals specifically dedicated to this subject were founded, treatises and important editions of classic authors were published, and a few universities launched the first courses on history of mathematics. Historians of mathematics such as Hieronymus George Zeuthen, Florian Cajori and Gino Loria supported the use of history in teacher education. The reasons for promoting this use were not only to foster the cultural enrichment derived from knowing something about the development of mathematical thinking, but also to provide a means suitable to revisit elementary mathematics from an advanced standpoint.

Around the end of the nineteenth century the way of looking at the history of mathematics was also influenced by the developments of biology, which were introducing new views in the scientific world. One of the most famous manifestations of this influence was the transposition of the biological law of recapitulation to the psychological development. According to this law, which may be summarised by the motto "ontogenesis recapitulates phylogenesis", the claim that in their intellectual development students naturally traverse more or less the same stages as mankind has been taken as a guarantee (sometimes implicitly) to ensure the link between the domains of history and of mathematics education. This view is echoed in the writings of some mathematicians. According to Klein ([12], p. 248), "the scholar must naturally follow the same course of development on a smaller scale, that the science itself has taken on a larger." Poincaré writes:

> "The educators' task is to make children follow the path that was followed by their fathers, passing quickly through certain stages without eliminating any of them. In this way, the history of sciences has to be our guide." ([13], p. 159, my translation)

The claims of these mathematicians have to be set in the context of the debate of their times about mathematical invention/discovery, and, in particular, about the role of intuition and rigour. In successive years the recapitulation law has been subject to deep revision, because of the developments of biological studies and the emergence of new concepts about the role of culture in the way we come to know and think (see [11], [14]).

These different kinds of stimuli (flourishing of historical studies, biological research and mathematicians' contributions to the debate of foundation of mathematics) influenced the school world. Around the end of the nineteenth century and the beginning of the twentieth we find works of mathematics education advocating the use of history in mathematics teaching, and documents which report on this use in classroom practice. Particularly remarkable is the paper [1], by a teacher of the Training Department of Alexandra College (Dublin), who introduced girls aged sixteen and seventeen to "a little of the story of mathematical growth" ([1], p. 72). In this report there is a plan for putting into practice the parallelism between the mathematics curriculum and the historical development of mathematics, in accordance with the ideas presented in Benchara Branford's book *A Study of Mathematical Education* (1908). Another example of the presence of history in the classroom is offered by an Italian mathematics teacher, who founded in 1895 and edited for about 20 years a mathematics journal for secondary students called *Il Pitagora* with the aim of attracting young people towards

mathematics. In presenting the journal the editor wrote explicitly that one of the main means of making mathematics attractive would be the history of mathematics (games were another means mentioned). The journal, indeed, published historical anecdotes, excerpts of ancient works and historical notes on elementary mathematical subjects such as arithmetic and geometry. Most authors of the articles were secondary teachers. What they wrote shows that in choosing the historical materials they actually cared for their students' needs. The historical aspect of this journal is analysed in [10].

In the past mathematics education was not developed as a scientific discipline and experiments (if any) were not reported and analysed according to modern methods. At present the development of mathematics education not only has changed the way of looking at the history of mathematics, but also provides the means for evaluating its use in teaching and learning from a scientific point of view. The International Study Group on the Relations between the History and Pedagogy of Mathematics was in fact one of the first study groups (together with the group of Psychology of Mathematics Education) affiliated to ICMI (this happened in 1976). Since then, many activities have provided materials (mainly historical documents and reports of experiments) which constitute the background to studying the role of the history of mathematics in the teaching and learning of mathematics. In the early 1980s the Summer Universities on the History and Epistemology in Mathematics Education were organised by the French Mathematics Education community with the French IREMs (*Instituts de Recherche sur l'Enseignement des Mathématiques*). In the UK the HIMED (History In Mathematics EDucation) conferences started in 1990; the British Society for the History of Mathematics was the organiser. A selection of the French papers documenting the experiments with the use of history in the classroom was translated into English, see [8]. In all these initiatives school teachers were active participants, as well as historians of mathematics and mathematics educators.

We may say that the theme of ICMI Study 10 is at the crossroads of different streams of research, the main ones being the history of mathematics, mathematics, pedagogy, and epistemology. The discussion document for the Study was elaborated by scholars belonging to these different streams and touched the main points of the theme at issue:

- why, how and when to use history in mathematics teaching;
- strategies to be used for the effective use of history at school, university and in teacher preparation;
- the state of the art (in research and in practice).

The discussion document attracted preparatory papers that were distributed to the participants so as to arouse critical reflections in the subgroups involved in the meeting. Each subgroup had one or more coordinators, but the atmosphere was such that the activities and the resulting book were very democratic, since all participants were given the opportunity to express their opinions, to report their experience and to record their interventions in the book edited by John Fauvel and Jan van Maanen [9] as a result of ICMI Study 10. The 62 contributors listed at the end of this book came from 26 countries of the five continents. Some of them were involved in tertiary education, mostly in the field of mathematics education. There was a minority of secondary teachers, but no teachers from primary schools. There is a remarkable isomorphism between the activities in the meeting and the resulting book, so that the analysis of the latter reflects what happened in the former.

## 2. ICMI Study 10 as a bridge between research and practice

One of the problems of ICMI activities is the multifaceted nature of the potential audience. Different communities with their specific needs and competencies compose the community of ICMI: mathematicians, curriculum developers, mathematics teachers, researchers in mathematics education. The community of students is the silent component in the background of ICMI activities. It is expected that an ICMI Study will affect decisions of the members of these communities who have responsibilities in mathematical instruction. Since I take the component 'teacher' as a pivot of the process of implementation in the classroom of whatever innovation, I deem that it is particularly interesting to investigate what impact ICMI Study 10 will have among teachers. Chapter 4 ("History of mathematics for trainee teachers") provides valid reasons to use the history of mathematics in teacher training courses and I myself have experienced this. After the publication of the ICMI Study book, when I carried out courses for in-service secondary school teachers I decided to use this book and gave it directly into participants' hands. My aim was twofold: on one hand to foster the reflection on the problems of mathematics teaching from a different point of view, and to provide materials for planning teaching sequences; and on the other hand it was my intention to see the effect of the ICMI Study in school practice. I outline some of the feedback in what follows.

At a first glance the chapter that most attracted the participants was that titled "Historical support for particular subjects" (Chapter 8). It provides a repertoire of historical materials suitable for developing the topics of mathematics treated in school: algebra, analysis, numbers and numerical systems, trigonometry, probability. This chapter evidences a powerful character of the ICMI Studies, that of bringing together different scholars from all around the world who represent different cultures. Since the history of mathematics is embedded in various cultures this emerged specifically in ICMI Study 10. The teachers attending my courses became aware of the multicultural character of mathematics and of the multifaceted aspects of human culture. In this way they were led to reflect on the meaning of 'humanistic mathematics' in teaching, by taking into account both senses discussed in [2] of "teaching humanistic mathematics" and "teaching mathematics humanistically." The theme of multiculturality pervades all this ICMI Study, as evidenced in Chapters 6 ("History in support of diverse educational requirements – opportunities for change") and 7 ("Integrating the history of mathematics in the classroom: an analytical survey"). In elaborating all these chapters the presence of secondary teachers who brought to the working groups their first-hand experience in different contexts was important.

The chapters mentioned earlier provided teachers with good examples of *how* history can be used in the classroom. The next step was to reflect on *why* the use of history can be an efficient means in the construction of mathematical knowledge, or, in other words, to identify reasons why the history of mathematics can meet the pedagogical goals characterising mathematics teaching. The development of this step is in the chapters which provide the theoretical background to the use of history in the teaching and learning of mathematics. This background encompasses research in mathematics education, epistemology, history of mathematics, and ethnomathematics. Chapter 2 ("Philosophical, multicultural and interdisciplinary issues") focuses on the discussion about the nature of mathematics and the embedding of mathematics in cultures. The history of mathematics offers hints to deepen these aspects and reveals itself as a good reason to go across disciplines.

Chapter 3 ("Integrating history: research perspectives") deals with the problem of judging the effectiveness of history in mathematics teaching. According to the authors of this chapter the techniques of quantitative experimental research are not suitable to this aim, instead qualitative research paradigms such as those developed in anthropology and ethnography may tell us if the introduction of history in mathematics teaching works. In this chapter nine articles written by teachers who have experienced the use of history are analysed in order to study its positive effects. In an article the genetic approach to calculus is considered; this brings to the fore a theme that pervades the book: the relation between the development of concepts in students' minds and in the history of mankind. In particular, this relation is analysed in Chapter 5 ("Historical formation and students' understanding of mathematics"). Studying this relation requires a clear epistemological approach which involves interpretations of students' understanding of mathematics and explanations of the history of mathematics. One of the themes faced in this chapter is the law of recapitulation that we mentioned at the beginning. It is recognised that the relation between ontogenesis and phylogenesis presents problems that its ancient supporters have neglected: for example, the links between knowledge and the context are underestimated, and historical facts are viewed from our modern point of view. Referring to my courses for teachers I have observed that the reflection on the issues faced in this chapter was very suitable to help the participants gain an appropriate understanding of relations between knowledge and historical development.

The discussion of the formation of mathematical concepts brings to the fore the need for using primary sources to exploit the power of history in mathematics teaching at its best. In Chapter 9 ("The use of original sources in mathematics classroom") this need is justified through reasons that I summarise as follows. Usually in schools mathematical concepts are presented in a polished way, which is satisfying from a mathematical point of view, but may be meaningless for students. To recover the meaning it may be useful to go back to the original words of the mathematicians who have conceived such concepts. In their very words the cognitive roots of the mathematical concepts, e.g. the key ideas around which the concepts have grown up, are made evident. This way of looking at the use of history applies also to teacher education: the primary sources may help teachers to reshape their conception of mathematics teaching by promoting the reflection on the nature of mathematical concepts and their genesis. Having said that, I must point out that the use of primary sources is not an easy task. Firstly, for school teachers without university contacts, there is the practical problem of the access to primary sources. Afterwards there is the problem of understanding them: primary sources are often written in foreign or dead languages, and the very words of ancient authors are subject to interpretation. Once this preliminary work has been done, there is the need of choosing the primary sources in such a way that they fulfil the educational needs of the classroom. As Burn puts it, "the selection of primary sources is of critical importance; badly chosen historical material may be as inaccessible as the most abstract mathematics" ([3], p. 212).

In the ICMI Study book the theoretical and the practical considerations previously mentioned are supported by rich lists of references at the end of each chapter. Moreover, the final Chapter 11 ("Bibliography for further work in the area") is an annotated bibliography of publications in different countries. Equally useful from the informative point of view is Chapter 10 ("Non-standard media and other resources"). This chapter sets a bridge between traditional ways of communicating mathematical culture and new ways: dramatisation, exhibitions, software, ancient mathematical instruments, surfing

the internet. It was particularly useful for the teachers attending the course to have information on the web sites that allow them to reach original sources and to visit science museums all around the world (cf. [5] and references therein). This technological dimension has changed the environment in which teachers face the work with history and provides them with some chance of using history successfully in their classroom. The importance of this chapter as a projection to new styles of teaching is stressed by the fact that the themes treated there are central in the discussion document of ICMI Study 16, whose focus is on investigating how mathematics education can satisfy the demand of making mathematics accessible to more people.

## 3. ICMI Study 10 in the classroom

The international survey of political issues provided by the first chapter ("The political context") shows that, in theory, no obstacles exist to the use of history in mathematics teaching; in many countries there is a tradition of looking at the history of mathematics as a means for widening the concept of mathematical culture. In some cases the use of history is advocated in order to recover a population's identity through the identification of the cultural mathematical heritage. In practice, the history of mathematics is absent from the university courses of many countries; the same happens in courses for teacher education. Obviously, teachers' lack of historical knowledge is a great obstacle to the use of history in teaching. Sometimes there is a contradiction between the indications by the Ministries of Education towards introducing a historical and epistemological approach into mathematics teaching and the scant freedom allowed teachers in dealing with official programmes. Thus these indications may be the source of problems or, more frequently, may be ignored.

There is a need for political action to make the mass of stimuli provided by the ICMI Study really applicable. For this reason not only did I use this ICMI Study regularly in the courses for teachers, but it was also my concern to see its outputs (if any) in the school environment. I observed that the Study was received with interest and that teachers were sometimes stimulated to look for books dealing with the history of mathematics on the shelves of bookshops. It was more difficult to find an actual application of the ideas discussed in the ICMI Study. The problems we hinted at in browsing the Study book (teachers not sufficiently confident in their historical knowledge, difficult access to primary sources, constraints of the school system) were making it difficult to realise the ideas put forward in the Study. It was difficult, but not impossible, as shown by the following example. A secondary teacher after having attended a course of mine organised a permanent working group with four colleagues. Together they collected primary sources suitable for grades 5 onwards in school. The historical materials were accompanied by proposals of activities in the classroom aimed at introducing a given mathematical topic or viewing it more critically. At the end of the work a book [6] containing these materials was edited. The same group of teachers has produced materials for primary schools based on the first printed book of mathematics *Larte de labbacho* (*Treviso Arithmetic*).

It is remarkable that this teacher has also taken into account the historical activities in assessing students. This fact evidences that history was not an optional activity, but was actually integrated into school teaching. The same teacher has also carried out another original experiment, which consisted of showing students historical pictures taken from ancient texts of mathematics accompanied by the following assignment:

- In our society, we communicate by using images: television, computer, posters and so on.
- Mathematics was born and developed in a social context. Many people have used it.
- Interpret the attached picture, i.e. look at the people in the picture and describe what you think they are doing. Examine the context and the details; try to use your mathematical knowledge.

What makes this task interesting is the fact that it focuses on a particular aspect of the history of mathematics, i.e. the charm and the informative value of the illustrations in antique historical sources. The central idea in the experiment is to use historical iconography for stimulating students' reflection on mathematical activity and its relation with social life. The report of the experiment was presented at a conference, see [7]. When I read this report it was natural for me to link the author's inspiration with something that the late John Fauvel liked a lot, e.g. the iconography in ancient mathematical books. The enjoyment in this aspect of history emerges in the ICMI Study book. It is moving for me to see in a teacher's work a tangible and spontaneous sign of Fauvel's cultural heritage.

## REFERENCES

[1] BARWELL, M. E. The advisability of including some instruction in the school course on the history of mathematics. *The Mathematical Gazette 7* (1913), 72–79.

[2] BROWN, S. I. Towards humanistic mathematics education. In: A. J. BISHOP et al. (eds), *International Handbook of Mathematics Education*, 1289–1321. Kluwer, Dordrecht–Boston–London, 1996.

[3] BURN, B. Book Review: History in Mathematics Education – The ICMI Study. *Educational Studies in Mathematics 52* (2003), 211–214.

[4] C.I.E.M. Discussion document for the tenth ICMI Study: "The role of the history of mathematics in the teaching and learning of mathematics". *L'Enseignement Math. (2) 43* (1997), 199–203.

[5] —— Discussion document for the sixteenth ICMI Study: "Challenging mathematics in and beyond the classroom". *L'Enseignement Math. (2) 51* (2005), 165–176.

[6] DEMATTÈ, A. *Fare matematica con i documenti storici.* IPRASE del Trentino, Trento, to appear.

[7] —— Not logically connected steps of reasoning: interpretation of historical mathematical pictures. In: *Proceedings of CIEAEM 57*, to appear.

[8] FAUVEL, J. (ed.) *History in the Mathematics Classroom.* The Mathematical Association, Leicester, 1990.

[9] FAUVEL, J. and J. VAN MAANEN (eds). *History in Mathematics Education: The ICMI Study.* (New ICMI Study Series, vol. 6.) Kluwer, Dordrecht–Boston–London, 2000.

[10] FURINGHETTI, F. The long tradition of history in mathematics teaching: an old Italian case. In V. KATZ (ed.), *Using History to Teach Mathematics: An International Perspective* (MAA Notes no. 51), 49–58. Math. Assoc. of America, 2000.

[11] FURINGHETTI, F. and L. RADFORD. Historical conceptual developments and the teaching of mathematics: rethinking phylogenesis and ontogenesis. In: L. ENGLISH (ed.) and M. BARTOLINI BUSSI, G. JONES, R. LESH & D. TIROSH (associated eds), *Handbook of International Research in Mathematics Education*, 631–654. L. Erlbaum Assoc., Mahwah (N.J.), 2002.

[12] KLEIN, F. The arithmetizing of mathematics. An address delivered at the public meeting of the Royal Academy of Sciences of Göttingen, November 2, 1895 (I. Maddison, trans.). *Bulletin of the Amer. Math. Soc. (II) 2* (1896), 241–249.

[13] POINCARÉ, H. La logique et l'intuition dans la science mathématique et dans l'enseignement. *L'Enseignement Math. 1* (1899), 157–162.

[14] SCHUBRING, G. Ontogeny and Phylogeny – Categories for Cognitive Development. In: F. FURINGHETTI, S. KAIJSER, C. TZANAKIS and A. VRETBLAD (eds), *Proceedings of the HPM Satellite Meeting and Fourth European Summer University* (second edition), to appear.

*(Reçu le 17 octobre 2005)*

Fulvia Furinghetti

Dipartimento di Matematica dell'Università di Genova
Via Dodecaneso 35
I-16146 Genova
Italy
*e-mail:* furinghe@dima.unige.it