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COMMISSION INTERNATIONALE DE L'ENSEIGNEMENT MATHÉMATIQUE

(THE INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION)

DISCUSSION DOCUMENT FOR THE SEVENTEENTH ICMI STUDY

DIGITAL TECHNOLOGIES AND MATHEMATICS TEACHING AND LEARNING: RETHINKING THE TERRAIN ("TECHNOLOGY REVISITED")

1. Introduction and rationale for ICMI Study 17

This document announces a new study to be conducted by the International Commission on Mathematical Instruction. The focus of this study, the seventeenth led by ICMI, will be the use of digital technologies in mathematics teaching and learning in countries across the world. ICMI Study 17 will seek to distinguish and address major theoretical challenges that face researchers working in this area, alongside practical challenges associated with implementing digital technologies in ways that fulfil their potential for teaching and learning mathematics.

In London in April 2004, the International Programme Committee (IPC) for ICMI Study 17 agreed upon three principles. First, it was decided that one major focus for ICMI Study 17 would be *cultural diversity* and how this diversity impinges on the use of digital technologies in mathematics teaching and learning, particularly in developing countries. Thus, the Study will specifically seek contributions from authors from developing countries and from those who offer a broad range of experiences. This orientation will illuminate our understanding of how far the expectations for digital technologies have been realised, in different phases of education, different countries, and different contexts within and outside educational institutions. We also judge that this focus will help the Study in the task of identifying and elaborating the resources needed to optimise the chances of sustained strategic change for the benefit of mathematics teaching and learning. Second, it was decided to delineate a set of *themes* to serve as the organising framework for the Study conference and for the subsequent ICMI publication. And third, the IPC agreed on a set of *approaches* through which participants in the Study will be invited to contribute to the themes.

This discussion document first presents the background to the Study, the challenges faced and the scope envisaged for the work. These sections then lead on to the descriptions of the seven organising themes of the Study and the five approaches that have been distinguished. Because the IPC anticipates that the Study conference will be organised around discussion within the themes (with some overarching sessions), each proposed contribution to the Study will be asked to state the theme into which it will fit. The IPC will also expect that any contribution to a theme would encompass a broad range of approaches. Finally the discussion document outlines the organisation, timing and location of the Study conference and the timetable of milestones leading up to the conference.

The first ICMI Study

This is the second time that an ICMI Study has centred on the use of computers in mathematics education. In fact, the first ever ICMI Study (undertaken in 1985) was entitled, "The Influence of Computers and Informatics on Mathematics and its Teaching". This Study represented one of the first attempts to develop a critical view of the role and influence of what was termed 'informatics' on mathematics education. The Study had a substantial impact, with the Study Volume first published by Cambridge University Press [1] but, once out of print, reissued in 1992 by UNESCO [2]. This latter volume included an overview written by Burkhardt and Fraser, which identified several questions on which future studies might focus: "Where are we going? Where do we want to go? Why? How do we know? How may we find out more? How do we get it to happen?" Authors also noted a major mismatch in timescale between the fast pace of change of technology, the slower changes in research in mathematics and the still slower changes in mathematics education itself and the curricula, particularly in relation to integrating technology into teaching and learning. Thus, in the seven years since the original 1985 meeting some aspects had moved swiftly while others had remained essentially static.

Some 20 years later, ICMI has felt the need to lead another critical reflection in this area. Consideration of the first Study provides an interesting starting point: for example, even a cursory glance reveals that the authors worked in a rather restricted set of countries (Europe and North America), and the focus of the papers was almost exclusively on using computers to model in an exploratory manner rather advanced mathematical ideas. Many authors pointed to the potential of using 'symbolic manipulators' in courses of calculus or linear algebra in order to allow students to focus on conceptual rather than procedural or technical issues.

It is worthwhile to note that despite the fact that many authors identified the potential of the systems they described, many also observed that there was little evidence of any significant impact on the mathematics curriculum of secondary schools and universities (primary-level mathematics was not considered). Reasons put forward included mathematicians' lack of experience in using the systems and the absence of strategic approaches to change.

Digital technologies: Further developments and questions of impact

Since 1992, there have been substantive developments in digital technologies, both in terms of hardware and software, encompassing for instance, calculator technology and the use of the Internet, as well as computers of all types, and including digital technology widely used in society such as mobile phones and digital cameras. These developments together with associated software have potential implications for mathematics teaching and learning at all phases of education, and indeed outside the formal contexts of education. ICMI Study 17 seeks to take stock of these developments and assess their impact in the broadest terms. Our intention is that the interpretation of the term digital technology will be broad: encompassing for instance, calculator technology and the use of the Internet as well as computers of all types.

Alongside developments in digital technologies, there have been changes in the goals, objectives and orientation of studies using them, as well as a broadening of the perspectives, theoretical frameworks and methodologies adopted. Recently, it might be argued that the pace of technological development has increased still further. Digital technologies are becoming ever more ubiquitous and their influence touches most, if not all, education systems. In many countries, it is hard to conceive of a world without high-speed interactivity and connectivity.

All these developments have spawned an increasing range of studies, some focused on the impact of specific software, others looking more broadly at the interaction of teachers, students and technologies. As Hoyles and Noss claimed [4]: "there are major research issues for mathematics education that are shaping and being shaped by the issues confronting 'technologists'." How far these issues have been addressed and their potential realised for the improvement of mathematics teaching, learning and the curriculum, remains a subject of debate that will continue in ICMI Study 17.

Key challenges for ICMI Study 17

Through ICMI Study 17, we will seek to identify and analyse some of the challenges in mathematics teaching and learning, practically and theoretically, in the light of the use of digital technologies. Most digital technologies do not make explicit how they work or how they can be used in mathematics education. This means that taking account of their design, particularly in terms of implications for epistemology, is a central challenge. But, as we attempt to incorporate new technological tools into teaching and learning, we also intend to make progress in trying to understand how the related epistemological structures are mediated by learning communities, and reciprocally, how learning communities are shaped by the artefacts and technologies in use.

In ICMI Study 17, we will not only seek to recognise the diversity in available software and hardware for use in mathematics education, but also consider seriously the influence of diverse curricula organisations, from highly centralised to locally autonomous, and of the availability of resources in different countries – whether it is access to handheld devices, computers or to the web. ICMI Study 17 will also seek to take account of cultural diversity and how issues of culture alongside those related to teacher beliefs and practice all shape the way digital technologies are used and their impact on mathematics and its teaching and learning.

2. Scope of the Study

Digital technologies are evolving quickly and pervade more and more aspects of social life, human thinking and knowledge. Twenty years after the first ICMI Study, the 'stand-alone' computer is no longer a paradigmatic tool and citizens experience everyday a variety of computerised tools (from small electronic appliances, more or less sophisticated calculators, mobile communication devices to worldwide networks). All this hardware is accompanied by an ever growing diversity of software: from applets that most people can build and use, to large applications designed and implemented by large transnational teams, few of which have educational goals.

In this varied and evolving context, ICMI Study 17 will seek a balance between two, potentially contradicting, aims:

- to reflect on *actual* uses of technology in mathematics education, avoiding mere speculation on hypothetical prospects,
- to address the range of hardware and software with a *potential* to impact upon or contribute to mathematics teaching and learning.

It is anticipated that this balance will be achieved through the elaboration of themes by which the Study will be organised, and by specifically encouraging contributions from diverse cultures with differing experiences, most notably from developing countries. It is timely to evaluate ideas and new prospects offered by tools as well as provide updated analyses. In addition, to complement the current body of knowledge, we need to address new developments, involving for example connectivity, and their impact on and contribution to mathematics epistemology and mathematics teaching and learning.

Mathematics Education itself has evolved since the first Study. Epistemological studies of mathematics and psychological approaches to learning mathematics are extensively supplemented by research within primary, secondary and university mathematics contexts as well as contexts outside of educational institutions. Issues around the teacher and classroom practices have been widely addressed using a number of theoretical frameworks, some specific to mathematics, some more general. The Study will aim to consider mathematics education in all these diverse and complementary aspects.

While we noted that the first Study was largely focused on modelling mathematics, more recently work has focused much more generally on the multitude of ways technology can shape teaching and learning mathematics, while reciprocally being shaped by its use. For example, studies have looked at the complex process of instrumental genesis, the role of the teacher and the connection of tool use and traditional techniques (cf. [3]). New robust paradigms for thinking about tool use in the context of mathematics education are beginning to emerge and ICMI Study 17 aims to take a further step forward in this direction.

We now turn to descriptions of the seven themes around which ICMI Study 17 will be organised.

3. The themes of ICMI Study 17

The ICMI Study will be organised around seven themes that provide complementary perspectives on the use of digital technologies in mathematics teaching and learning. Contributions to the separate themes will be distinguished by the theme's specific foci and questions, although each contribution is expected to encompass a broad range of approaches.

The seven themes are: Mathematics and mathematical practices; Learning and assessing mathematics with and through digital technologies; Teachers and teaching; Designing learning environments and curricula; Implementation in curricula and in classrooms; Access, equity and socio-cultural issues; Connected and networked classrooms.

1. Mathematics and mathematical practices

This theme will address the relationship between the changes in both mathematics and mathematical practices that digital technologies have precipitated or enabled. We are currently witnessing a shift in the ways that mathematicians work on problems – for example, the use of an experimental methodology – and in the kinds of problems they are choosing to pursue, a shift that has been influenced by the presence and power of digital technologies. Some have argued that these shifts in knowledge and practice are accompanied by a shift in values, as can be seen, for instance, in the growth and acceptance of computer-generated results.

Possible questions:

- What new types of mathematical knowledge and practices emerge as a result of access to digital technologies, particularly computational, dynamic visualisation and communication technologies?
- How are new types of technology-mediated mathematical knowledge and practices related to current classroom curricula and values, what curriculum elements exist because older-than-digital technologies were all that was available, and how should aspects of mathematics curriculum therefore be deleted or changed?
- How are the changes in mathematics and mathematical practices affecting the communication of mathematics across diverse cultures, and between university and school settings?
- How is access to digital technology impacting on the mathematical practices that occur outside of formal education settings?
- How can a focus on the role of technological tools help us understand the ways in which mathematical practices vary across settings?
- What role can the "mathematics laboratory" play in different educational contexts, including primary, secondary, tertiary and vocational education?

2. Learning and assessing mathematics with and through digital technologies

This theme will concentrate on developing understanding of how students learn mathematics with digital technologies and the implications of the integration of technological tools into mathematics teaching for assessment practices. Its foci will include consideration of how digital technologies might be employed to open windows on learners' developing knowledge, and on how interactions with digital tools mediate learning trajectories. Additionally, the theme will address the challenges involved in balancing use of mental, paper-and-pencil, and digital tools in both assessment and teaching activities.

Possible questions:

- What theoretical approaches and methodologies help to illuminate students' learning of mathematics in technology-integrated environments? What are the relationships between these approaches and how do they compare or contrast with other theories of mathematics learning?
- How does the use of different digital technologies influence the learning of different mathematical concepts and the shape of the trajectories through which the learning develops?
- How can technology-integrated environments be designed so as to capture significant moments of learning?
- How can the assessment of students' mathematical learning be designed to take into account the integration of digital technologies and the ways that digital technologies might have been used in the learning of mathematics?
- How can and should learning and assessment practices reflect differences in resource level and in cultural heritage?
- How can the benefits of existing technologies be maximised for the benefit of mathematics teaching and learning?

3. Teachers and teaching

The integration of any new artefact into a teaching situation can be expected to alter its existing equilibrium and requires teachers to undergo a complex process of adaptation. In the case of digital technologies, the modifications required of routine practices are likely to be particularly pronounced. Not only might different pedagogical approaches be appropriate, but also the teacher needs to reconsider how the new representations and alternative learning strategies made available through technology use might change along with what could be taught, and how and when. Various frameworks, drawing from both theory and practice, are currently employed to analyse the role of the teacher in orchestrating technology-integrated mathematics learning. This theme will consider the complementarities and contrasts between these frameworks and how they are operationalised in the face of ever-evolving resources. It will also address implications of these complex issues for pre-service and for ongoing teacher professional development.

POSSIBLE QUESTIONS:

- What theoretical frameworks and methodologies illuminate the teacher's role in technology-integrated environments for mathematics learning?
- What kinds of pedagogical approaches and classroom organisations can be employed in technology-integrated environments and how can they be evaluated?
- How can teachers be supported in deciding why, when and how to implement technological resources into their teaching practices?
- What kinds of pre-service education and professional development programs are appropriate to prepare teachers to use technology in their mathematics classrooms and to help them to sustain ongoing use?
- What can we learn from teachers who use, or who have tried to use, digital technologies for mathematics teaching?
- How are teachers' beliefs, attitudes, mathematical and pedagogical knowledge shaped and reshaped by their use of digital technologies in mathematics teaching and how are these issues influenced by access to resources and by differences in culture?

4. DESIGN OF LEARNING ENVIRONMENTS AND CURRICULA

The purpose of this theme is to focus closely on the issues and challenges involved in designing mathematics learning environments that integrate digital technologies while recognising that the tools made available in such environments can and do shape mathematical activity in ways that to some extent are predictable and in some not. In addition to considering the specific affordances and constraints of different digital technologies for structuring mathematical learning experiences (including various software packages, hardware configurations and the Internet), this theme will consider the implications of design decisions on tools, curriculum, teaching and learning.

POSSIBLE QUESTIONS:

- What theoretical frameworks and methodologies are helpful in understanding how design issues impact upon the teaching and learning of mathematics?
- What does it mean to be mathematically literate, in a world instrumented by technology?
- What kinds of mathematical activities might different digital technologies afford and how can learning experiences (including the tools, the tasks and the settings) be designed to take advantage of these affordances?
- How can technology-integrated learning environments be designed so as to influence and change curriculum, and how can this be achieved consistently over time?
- How can digital tools be designed to foster mathematical thinking? What general design principles might be articulated that inform the development of technological tools for mathematics education? How should these design principles be adapted for web-based mathematics learning either face-to-face or at a distance?
- How can technology-based learning environments be designed so as to remain sensitive to persistent challenges, for example swift and inevitable obsolescence and ongoing maintenance costs?

5. Implementation of curricula and classroom practice

Since the first ICMI Study, developments in digital technology have resulted in a range of applications for mathematics and mathematics teaching and learning. Furthermore, governments have developed policies to promote the learning and use of digital technologies throughout education systems in general as well as for mathematics learning in particular. Thus there has been some systemic implementation of digital technologies in mathematics education as a result of policy initiatives, alongside more scattered implementation as a result of specific innovations and initiatives. Neither centralised nor local initiatives have tended to result in widespread and sustained use in mathematics curricula and in teaching. The influence and place of digital technology at all levels of mathematics education provides a unique opportunity to examine reform and change in mathematics curricula and teaching by, for example, examining the political and social forces that promote or impede any integration of digital technologies in mathematics. Issues related to scaling initiatives and the challenges of systemic change will be explored in this theme.

Possible questions:

- What theoretical frameworks and methodologies are helpful in understanding issues related to the widespread implementation of digital technologies in mathematics education ?
- How have mathematics curricula changed to reflect developments in mathematics afforded by digital technologies ?
- How have countries with different economic capacity or with different cultural heritage and practices implemented digital technologies in mathematics education ?
- What have we learned about the process of change and reform in mathematics education through our successful and unsuccessful experiences of implementing digital technologies in mathematics education ?
- What approaches, strategies or factors foster or impede the implementation of technology-rich mathematics education ? What issues are involved for policy-makers, administrators and teachers in the organisation of technology resources in educational settings ?
- How can students, teachers and schools who have access to modest technologies, or limited access to cutting-edge versions of technology, be supported to use the technology effectively ?
- What are the major issues of scale up, the different phases of scale up, and how can technologies be modified, designed, and used to facilitate diffusion of technological (and other) innovations ?

6. Access, equity and socio-cultural issues

Access to and use of digital technologies differ between countries and within countries according to socio-economic, gender and cultural factors. In this theme, the aim will be to explore how political, economic, social and cultural factors influence access to digital technologies for mathematics and quality learning in technology-integrated mathematics. We will also seek to understand how cultural practices in technology-integrated mathematics have enhanced, or eroded, issues of equity and agency in mathematics education.

POSSIBLE QUESTIONS:

- What are the influences of globalisation on mathematics education and the use of technology? How does mathematics education respond productively to globalisation issues?
- How and to what extent has the use of digital technologies in mathematics education enabled, or eroded, equity and agency in mathematics education?
- How and to what extent has technology-integrated mathematics contributed to, or reduced, differences between countries in participation and achievement in mathematics?
- What can students and teachers with limited access to digital technologies, or access to modest technologies for mathematics learning do with technology that is empowering for students?
- How can the use of digital technologies in mathematics education support the learning of students with special needs?
- How can digital technologies be used in mathematics learning to respond to the diverse needs of all learners, regardless of mathematics achievement, sex, class, ethnicity or cultural background?

7. CONNECTIVITY AND VIRTUAL NETWORKS FOR LEARNING

Digital technologies have already changed the ways we think about interacting with mathematical objects, especially in terms of dynamic visualisations and the multiple connections that can be made between different kinds of symbolic representation. At the same time, we are seeing rapid developments in the ways that it is possible to interact and collaborate through technological devices. In this theme, the potential and challenges for mathematics education of the increasing levels of connectivity, both within and between classrooms, will be considered.

POSSIBLE QUESTIONS:

- How can theoretical frameworks and methodologies developed for interpreting activity, learning and teaching in technology-integrated classrooms be extended to assist in understanding the distance-learning context? What kinds of changes and refinements are needed?
- What is the potential contribution to mathematics learning of different levels of interactivity and different modalities of interaction, and how might this potential be realised?
- What is special about the potential of collaborative study of mathematics whilst physically separated, and how might this potential be harnessed so as to support mathematics learning?
- What is the potential for creating virtual communities for mathematics learning and permitting communication between individuals from different educational settings?

4. Approaches to the themes

Different and complementary approaches to the themes of ICMI Study 17 are expected to follow from the diversity in its scope. Our broad classification distinguishes five approaches. Contributions to any theme will be expected to take seriously more than one approach; one approach might define the choice of theme, while other complementary approaches ensure a rounded contribution.

1. Impact on mathematics

Technology modifies mathematical practices in research, influences the development of new mathematical domains and changes the relationship between mathematics and society. Thus, mathematics, its different facets and domains, and their evolution are central to this approach.

2. Roles of different digital technologies

The Study cannot ignore the diversity of technological tools now available. Contributions should discuss the influence of a given technology on research findings, and make appropriate comparisons (for example, between programmable microworlds and expressive tools (see [4]), between handhelds and more traditional computer use, between face-to-face or distance teaching and learning or learning through the web). Technological tools however should not to be taken as given and considerations of curriculum and classroom practices should help to examine a tool's design in view of approaches to teaching and learning.

3. Contribution to learning mathematics

This approach brings a focus on questions such as how does a human being learn mathematics with technology and what mathematics does (s)he learn. These questions could be addressed in terms of cognition or affect, with regard to mathematical fields, activities and contexts at different school levels, or in contexts in and out of school.

4. The role of the teacher

Some consideration of the teacher is essential to developing any understanding of the integration of digital technologies in classrooms. Investigation can take different forms. It might involve observation and analyses of effective uses of digital technologies by teachers in naturalistic settings; interventionist design experiments, "didactical engineering", that implement and evaluate new curricula and teacher practice; or comparative analyses in programmes of teacher education of different approaches to the use of digital technologies in terms of their effectiveness and impact.

5. Theoretical frameworks

There is a diversity of theoretical frameworks in technology and mathematics education, some inspired by more general conceptualizations and others concerning paradigms for thinking about tool use. Contributions to the Study based on empirical research should seek to make explicit their theoretical foundations and the implications of these frameworks. More theoretically oriented contributions might work on convergences between frameworks. Alongside these different theories are also different paradigms of research that must be mobilised to address a range of research questions and to achieve a range of different outcomes.

5. The Study Conference

ICMI Study 17 is designed to enable researchers and practitioners around the world to share research, theoretical work, project descriptions and analyses. Although research papers will form part of the program, substantial time will also be allocated for direct engagement with software and reports of practice, and for collective work on significant problems in the field, that will eventually form part of the publication from the Study.

We anticipate that participants will be organised into working groups around the Themes and that these groups will meet regularly during the time of the Conference. In each working group, the IPC will organise the discussion starting from the contributions and offer participants the opportunity to demonstrate specific software applications, teaching materials or data related to their contribution. Thus, there will plenty of time for discussion of submitted papers, as well as possible plans for future collaborative activity. We will experiment with useful formats for the exchange of ideas generated in the course of the Study Conference.

Location and Dates. The Study Conference will take place in Hanoi, Vietnam and will be hosted by the Hanoi Institute of Technology from 3–8 December 2006 (starting on the evening of 3 Dec. and ending at lunchtime on 8 Dec.). Every effort will be made to assist participants with visa applications.

Participation. As is the normal practice for ICMI studies, participation in the Study conference will be by invitation only, given on the basis of a submitted contribution. Proposed contributions will be reviewed and selections made according to the quality of the work, the potential to contribute to the advancement of the Study, with explicit links to the themes and approaches outlined in the Discussion Document, and the need to ensure diversity among the perspectives. The number of participants invited to participate will be limited to approximately 120 people.

ICMI Study 17 Book. The production of the ICMI Study 17 Book will be a *separate* exercise from the Study Conference. Clearly the volume will be informed by the papers and the discussions at the Study Conference as well as its outcomes, but it must be appreciated by all participants that there will be *no* guarantee that any of the papers accepted for the Study Conference will appear in the book. The Study Book will be an edited volume published by Springer as part of the New ICMI Study Series. The editors and the editing process and content will be the subject of discussion among the International Programme Committee and a framework prepared for the Study Conference.

It is expected that the organisation of the book will follow the organisation and themes of this Discussion Document. A report on the Study and its outcomes will be presented at the 11th International Congress on Mathematical Education to be held in Monterrey, Mexico, in 2008. It is hoped that the Study Book will also be published in 2008.

CALL FOR CONTRIBUTIONS TO ICMI STUDY 17

The International Programme Committee (IPC) for ICMI Study 17 invites submission of contributions on specific questions, problems or issues related to this Discussion Document. Contributions may be submitted, in the form of synopses of research papers or discussion papers that address one theme raised in the Discussion Document, with due reference to the approaches adopted. The International Programme Committee welcomes high-quality proposals from diverse researchers and practitioners who can make solid practical and scientific contributions to the Study. New researchers in the field are encouraged to submit proposals, as are those actively engaged in the development of technology in mathematics education. Researchers in mathematics are urged to submit proposals about changes in their practices or in the relationship between mathematics and society. To ensure a rich and varied scope of resources for the Study, participation from countries with different economic capacity or with different cultural heritage and practices is encouraged.

The contributions of those invited to the conference will be made available to other participants by way of conference pre-proceedings and on the ICMI Study website (http://www.math.msu.edu/~nathsinc/ICMI/). An invitation to the conference does not imply that a formal presentation of the submitted contribution will be made during the conference or that the paper will appear in the Study Volume published after the conference. Unfortunately, an invitation to participate in the conference does not imply financial support from the organisers, and participants should finance their own attendance at the conference. Funds are being sought to provide partial support to enable participants from non-affluent countries to attend the conference, but it is unlikely that more than a few such grants will be available.

Submissions should be between 6 and 8 pages (Times 14-point font, single spaced lines). Each must:

- include a 200 word abstract;
- make explicit the theme in which it is anticipated that the contribution will be discussed along with the approaches adopted;
- be written in English, the language of the conference.

Submissions should be made using the ICMI Study website and uploaded in the form of a PDF, RTF or DOC file not exceeding 1.5 MB. All submissions must be uploaded electronically at the ICMI Study website along with appropriate personal data.

Date of submission and review: no later than January 15, 2006, but earlier if possible.

Proposals will be reviewed and decisions made about inclusion in the Conference Program in May 2006 and notification of these decisions sent by mid May, 2006 to all those who submitted proposals.

Costs and details of accommodation will be available on the official website, which will be regularly updated with information about the Study and the Study Conference.

INTERNATIONAL PROGRAMME COMMITTEE

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The official website for the Study is http://www.math.msu.edu/~nathsinc/ICMI/

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COMMISSION INTERNATIONALE DE L'ENSEIGNEMENT MATHÉMATIQUE

(THE INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION)

A REPORT ON THE ICMI STUDY: "THE ROLE OF THE HISTORY OF MATHEMATICS IN THE TEACHING AND LEARNING OF MATHEMATICS"

by Fulvia FURINGHETTI *)

1. ICMI STUDY 10

ICMI Study 10 is entitled "The role of the history of mathematics in the teaching and learning of mathematics". The discussion document [4] initially appeared in 1997; the Study Conference took place at CIRM, the country retreat of the French Mathematical Society at Luminy (near Marseille, France) from 20 to 25 April 1998. The volume resulting from the Study and edited by the two co-chairs [9] was presented at ICME 9 in Japan (2000). The argument of this ICMI Study is far more wide-ranging than discussing the opportunity of teaching some elements of the history of mathematics in the mathematics courses of the various school levels and at university: it concerns the goals of mathematics teaching and the way to reach them through history. As the editors of the Study volume put it (p. xvii), the ICMI Study "is posited on the experience of many mathematics teachers across the world that its history makes a difference: that having the history of mathematics as a resource for the teacher is beneficial. School mathematics reflects the wider aspect of mathematics as a cultural activity."

*) Thanks to Peter Ransom for helping to revise the English of this paper.

The theme of ICMI Study 10 had never been considered in previous ICMI Studies, but the relation between mathematics and its history has interested the world of mathematics education for a long time. This happened especially around the end of the nineteenth and the beginning of the twentieth century. At those times the history of mathematics was developing as an autonomous discipline: journals specifically dedicated to this subject were founded, treatises and important editions of classic authors were published, and a few universities launched the first courses on history of mathematics. Historians of mathematics such as Hieronymus George Zeuthen, Florian Cajori and Gino Loria supported the use of history in teacher education. The reasons for promoting this use were not only to foster the cultural enrichment derived from knowing something about the development of mathematical thinking, but also to provide a means suitable to revisit elementary mathematics from an advanced standpoint.

Around the end of the nineteenth century the way of looking at the history of mathematics was also influenced by the developments of biology, which were introducing new views in the scientific world. One of the most famous manifestations of this influence was the transposition of the biological law of recapitulation to the psychological development. According to this law, which may be summarised by the motto "ontogenesis recapitulates phylogenesis", the claim that in their intellectual development students naturally traverse more or less the same stages as mankind has been taken as a guarantee (sometimes implicitly) to ensure the link between the domains of history and of mathematics education. This view is echoed in the writings of some mathematicians. According to Klein ([12], p. 248), "the scholar must naturally follow the same course of development on a smaller scale, that the science itself has taken on a larger." Poincaré writes:

> "The educators' task is to make children follow the path that was followed by their fathers, passing quickly through certain stages without eliminating any of them. In this way, the history of sciences has to be our guide." ([13], p. 159, my translation)

The claims of these mathematicians have to be set in the context of the debate of their times about mathematical invention/discovery, and, in particular, about the role of intuition and rigour. In successive years the recapitulation law has been subject to deep revision, because of the developments of biological studies and the emergence of new concepts about the role of culture in the way we come to know and think (see [11], [14]).

These different kinds of stimuli (flourishing of historical studies, biological research and mathematicians' contributions to the debate of foundation of mathematics) influenced the school world. Around the end of the nineteenth century and the beginning of the twentieth we find works of mathematics education advocating the use of history in mathematics teaching, and documents which report on this use in classroom practice. Particularly remarkable is the paper [1], by a teacher of the Training Department of Alexandra College (Dublin), who introduced girls aged sixteen and seventeen to "a little of the story of mathematical growth" ([1], p. 72). In this report there is a plan for putting into practice the parallelism between the mathematics curriculum and the historical development of mathematics, in accordance with the ideas presented in Benchara Branford's book *A Study of Mathematical Education* (1908). Another example of the presence of history in the classroom is offered by an Italian mathematics teacher, who founded in 1895 and edited for about 20 years a mathematics journal for secondary students called *Il Pitagora* with the aim of attracting young people towards mathematics. In presenting the journal the editor wrote explicitly that one of the main means of making mathematics attractive would be the history of mathematics (games were another means mentioned). The journal, indeed, published historical anecdotes, excerpts of ancient works and historical notes on elementary mathematical subjects such as arithmetic and geometry. Most authors of the articles were secondary teachers. What they wrote shows that in choosing the historical materials they actually cared for their students' needs. The historical aspect of this journal is analysed in [10].

In the past mathematics education was not developed as a scientific discipline and experiments (if any) were not reported and analysed according to modern methods. At present the development of mathematics education not only has changed the way of looking at the history of mathematics, but also provides the means for evaluating its use in teaching and learning from a scientific point of view. The International Study Group on the Relations between the History and Pedagogy of Mathematics was in fact one of the first study groups (together with the group of Psychology of Mathematics Education) affiliated to ICMI (this happened in 1976). Since then, many activities have provided materials (mainly historical documents and reports of experiments) which constitute the background to studying the role of the history of mathematics in the teaching and learning of mathematics. In the early 1980s the Summer Universities on the History and Epistemology in Mathematics Education were organised by the French Mathematics Education community with the French IREMs (*Instituts de Recherche sur l'Enseignement des Mathématiques*). In the UK the HIMED (History In Mathematics EDucation) conferences started in 1990; the British Society for the History of Mathematics was the organiser. A selection of the French papers documenting the experiments with the use of history in the classroom was translated into English, see [8]. In all these initiatives school teachers were active participants, as well as historians of mathematics and mathematics educators.

We may say that the theme of ICMI Study 10 is at the crossroads of different streams of research, the main ones being the history of mathematics, mathematics, pedagogy, and epistemology. The discussion document for the Study was elaborated by scholars belonging to these different streams and touched the main points of the theme at issue:

- why, how and when to use history in mathematics teaching;
- strategies to be used for the effective use of history at school, university and in teacher preparation;
- the state of the art (in research and in practice).

The discussion document attracted preparatory papers that were distributed to the participants so as to arouse critical reflections in the subgroups involved in the meeting. Each subgroup had one or more coordinators, but the atmosphere was such that the activities and the resulting book were very democratic, since all participants were given the opportunity to express their opinions, to report their experience and to record their interventions in the book edited by John Fauvel and Jan van Maanen [9] as a result of ICMI Study 10. The 62 contributors listed at the end of this book came from 26 countries of the five continents. Some of them were involved in tertiary education, mostly in the field of mathematics education. There was a minority of secondary teachers, but no teachers from primary schools. There is a remarkable isomorphism between the activities in the meeting and the resulting book, so that the analysis of the latter reflects what happened in the former.

2. ICMI Study 10 as a bridge between research and practice

One of the problems of ICMI activities is the multifaceted nature of the potential audience. Different communities with their specific needs and competencies compose the community of ICMI: mathematicians, curriculum developers, mathematics teachers, researchers in mathematics education. The community of students is the silent component in the background of ICMI activities. It is expected that an ICMI Study will affect decisions of the members of these communities who have responsibilities in mathematical instruction. Since I take the component 'teacher' as a pivot of the process of implementation in the classroom of whatever innovation, I deem that it is particularly interesting to investigate what impact ICMI Study 10 will have among teachers. Chapter 4 ("History of mathematics for trainee teachers") provides valid reasons to use the history of mathematics in teacher training courses and I myself have experienced this. After the publication of the ICMI Study book, when I carried out courses for in-service secondary school teachers I decided to use this book and gave it directly into participants' hands. My aim was twofold: on one hand to foster the reflection on the problems of mathematics teaching from a different point of view, and to provide materials for planning teaching sequences; and on the other hand it was my intention to see the effect of the ICMI Study in school practice. I outline some of the feedback in what follows.

At a first glance the chapter that most attracted the participants was that titled "Historical support for particular subjects" (Chapter 8). It provides a repertoire of historical materials suitable for developing the topics of mathematics treated in school: algebra, analysis, numbers and numerical systems, trigonometry, probability. This chapter evidences a powerful character of the ICMI Studies, that of bringing together different scholars from all around the world who represent different cultures. Since the history of mathematics is embedded in various cultures this emerged specifically in ICMI Study 10. The teachers attending my courses became aware of the multicultural character of mathematics and of the multifaceted aspects of human culture. In this way they were led to reflect on the meaning of 'humanistic mathematics' in teaching, by taking into account both senses discussed in [2] of "teaching humanistic mathematics" and "teaching mathematics humanistically." The theme of multiculturality pervades all this ICMI Study, as evidenced in Chapters 6 ("History in support of diverse educational requirements – opportunities for change") and 7 ("Integrating the history of mathematics in the classroom: an analytical survey"). In elaborating all these chapters the presence of secondary teachers who brought to the working groups their first-hand experience in different contexts was important.

The chapters mentioned earlier provided teachers with good examples of *how* history can be used in the classroom. The next step was to reflect on *why* the use of history can be an efficient means in the construction of mathematical knowledge, or, in other words, to identify reasons why the history of mathematics can meet the pedagogical goals characterising mathematics teaching. The development of this step is in the chapters which provide the theoretical background to the use of history in the teaching and learning of mathematics. This background encompasses research in mathematics education, epistemology, history of mathematics, and ethnomathematics. Chapter 2 ("Philosophical, multicultural and interdisciplinary issues") focuses on the discussion about the nature of mathematics and the embedding of mathematics in cultures. The history of mathematics offers hints to deepen these aspects and reveals itself as a good reason to go across disciplines.

Chapter 3 ("Integrating history: research perspectives") deals with the problem of judging the effectiveness of history in mathematics teaching. According to the authors of this chapter the techniques of quantitative experimental research are not suitable to this aim, instead qualitative research paradigms such as those developed in anthropology and ethnography may tell us if the introduction of history in mathematics teaching works. In this chapter nine articles written by teachers who have experienced the use of history are analysed in order to study its positive effects. In an article the genetic approach to calculus is considered; this brings to the fore a theme that pervades the book: the relation between the development of concepts in students' minds and in the history of mankind. In particular, this relation is analysed in Chapter 5 ("Historical formation and students' understanding of mathematics"). Studying this relation requires a clear epistemological approach which involves interpretations of students' understanding of mathematics and explanations of the history of mathematics. One of the themes faced in this chapter is the law of recapitulation that we mentioned at the beginning. It is recognised that the relation between ontogenesis and phylogenesis presents problems that its ancient supporters have neglected: for example, the links between knowledge and the context are underestimated, and historical facts are viewed from our modern point of view. Referring to my courses for teachers I have observed that the reflection on the issues faced in this chapter was very suitable to help the participants gain an appropriate understanding of relations between knowledge and historical development.

The discussion of the formation of mathematical concepts brings to the fore the need for using primary sources to exploit the power of history in mathematics teaching at its best. In Chapter 9 ("The use of original sources in mathematics classroom") this need is justified through reasons that I summarise as follows. Usually in schools mathematical concepts are presented in a polished way, which is satisfying from a mathematical point of view, but may be meaningless for students. To recover the meaning it may be useful to go back to the original words of the mathematicians who have conceived such concepts. In their very words the cognitive roots of the mathematical concepts, e.g. the key ideas around which the concepts have grown up, are made evident. This way of looking at the use of history applies also to teacher education: the primary sources may help teachers to reshape their conception of mathematics teaching by promoting the reflection on the nature of mathematical concepts and their genesis. Having said that, I must point out that the use of primary sources is not an easy task. Firstly, for school teachers without university contacts, there is the practical problem of the access to primary sources. Afterwards there is the problem of understanding them: primary sources are often written in foreign or dead languages, and the very words of ancient authors are subject to interpretation. Once this preliminary work has been done, there is the need of choosing the primary sources in such a way that they fulfil the educational needs of the classroom. As Burn puts it, "the selection of primary sources is of critical importance; badly chosen historical material may be as inaccessible as the most abstract mathematics" ([3], p. 212).

In the ICMI Study book the theoretical and the practical considerations previously mentioned are supported by rich lists of references at the end of each chapter. Moreover, the final Chapter 11 ("Bibliography for further work in the area") is an annotated bibliography of publications in different countries. Equally useful from the informative point of view is Chapter 10 ("Non-standard media and other resources"). This chapter sets a bridge between traditional ways of communicating mathematical culture and new ways: dramatisation, exhibitions, software, ancient mathematical instruments, surfing the internet. It was particularly useful for the teachers attending the course to have information on the web sites that allow them to reach original sources and to visit science museums all around the world (cf. [5] and references therein). This technological dimension has changed the environment in which teachers face the work with history and provides them with some chance of using history successfully in their classroom. The importance of this chapter as a projection to new styles of teaching is stressed by the fact that the themes treated there are central in the discussion document of ICMI Study 16, whose focus is on investigating how mathematics education can satisfy the demand of making mathematics accessible to more people.

3. ICMI Study 10 in the classroom

The international survey of political issues provided by the first chapter ("The political context") shows that, in theory, no obstacles exist to the use of history in mathematics teaching; in many countries there is a tradition of looking at the history of mathematics as a means for widening the concept of mathematical culture. In some cases the use of history is advocated in order to recover a population's identity through the identification of the cultural mathematical heritage. In practice, the history of mathematics is absent from the university courses of many countries; the same happens in courses for teacher education. Obviously, teachers' lack of historical knowledge is a great obstacle to the use of history in teaching. Sometimes there is a contradiction between the indications by the Ministries of Education towards introducing a historical and epistemological approach into mathematics teaching and the scant freedom allowed teachers in dealing with official programmes. Thus these indications may be the source of problems or, more frequently, may be ignored.

There is a need for political action to make the mass of stimuli provided by the ICMI Study really applicable. For this reason not only did I use this ICMI Study regularly in the courses for teachers, but it was also my concern to see its outputs (if any) in the school environment. I observed that the Study was received with interest and that teachers were sometimes stimulated to look for books dealing with the history of mathematics on the shelves of bookshops. It was more difficult to find an actual application of the ideas discussed in the ICMI Study. The problems we hinted at in browsing the Study book (teachers not sufficiently confident in their historical knowledge, difficult access to primary sources, constraints of the school system) were making it difficult to realise the ideas put forward in the Study. It was difficult, but not impossible, as shown by the following example. A secondary teacher after having attended a course of mine organised a permanent working group with four colleagues. Together they collected primary sources suitable for grades 5 onwards in school. The historical materials were accompanied by proposals of activities in the classroom aimed at introducing a given mathematical topic or viewing it more critically. At the end of the work a book [6] containing these materials was edited. The same group of teachers has produced materials for primary schools based on the first printed book of mathematics *Larte de labbacho* (*Treviso Arithmetic*).

It is remarkable that this teacher has also taken into account the historical activities in assessing students. This fact evidences that history was not an optional activity, but was actually integrated into school teaching. The same teacher has also carried out another original experiment, which consisted of showing students historical pictures taken from ancient texts of mathematics accompanied by the following assignment:

- In our society, we communicate by using images: television, computer, posters and so on.
- Mathematics was born and developed in a social context. Many people have used it.
- Interpret the attached picture, i.e. look at the people in the picture and describe what you think they are doing. Examine the context and the details; try to use your mathematical knowledge.

What makes this task interesting is the fact that it focuses on a particular aspect of the history of mathematics, i.e. the charm and the informative value of the illustrations in antique historical sources. The central idea in the experiment is to use historical iconography for stimulating students' reflection on mathematical activity and its relation with social life. The report of the experiment was presented at a conference, see [7]. When I read this report it was natural for me to link the author's inspiration with something that the late John Fauvel liked a lot, e.g. the iconography in ancient mathematical books. The enjoyment in this aspect of history emerges in the ICMI Study book. It is moving for me to see in a teacher's work a tangible and spontaneous sign of Fauvel's cultural heritage.

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